الجامعة الأردنيـــــة
كلية الدراسات العليا

التاريـــخ: / /

نموذج رقم (١٨)
اقرار والتزام بالمعايير الأخلاقية والأمانة العلمية
وقوانين الجامعة الأردنية وأنظمتها وتعليماتها
لطلبة الماجستير

أنا الطالب: حلا عبدالله محمد عارف   الرقم الجامعي: (٨٠٧٠٤٨٣)
تخصص: الهندسة الصناعية/ادارة   الكليـــــة: الهندسة والتكنولوجيا

عنوان الرسالة: Q,R Model for deteriorating Inventory with delay in payment & Backorder cancellation

اعلن بأنني قد التزمت بقوانين الجامعة الأردنية وأنظمتها وتعليماتها وقراراتها السارية المفعول المتعلقة باعداد رسائل الماجستير عندما قمت شخصيا" باعداد رسالتي وذلك بما ينسجم مع الأمانة العلمية وكافة المعايير الأخلاقية المتعارف عليها في كتابة الرسائل العلمية. كما أنني أعلن بأن رسالتي هذه غير منقولة أو مستلة من رسائل أو كتب أو أبحاث أو أي منشورات علمية تم نشرها أو تخزينها في أي وسيلة اعلامية، وتأسيسا" على ما تقدم فانني أتحمل المسؤولية بأنواعها كافة فيما لو تبين غير ذلك بما فيه حق مجلس العمداء في الجامعة الأردنية بالغاء قرار منحي الدرجة العلمية التي حصلت عليها وسحب شهادة التخرج مني بعد صدورها دون أن يكون لي أي حق في التظلم أو الاعتراض أو الطعن بأي صورة كانت في القرار الصادر عن مجلس العمداء بهذا الصدد.

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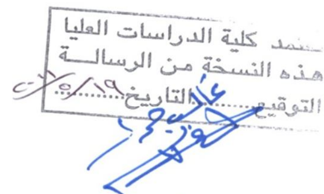
Q,R MODEL FOR DETERIORATING INVENTORY WITH DELAY IN PAYMENT AND BACK ORDER CANCELLATION

By
Dima Abdalla Ayesh

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Dr. Mahmoud Barghash

This Thesis was Submitted in Partial Fulfillment of the Requirements for the Master's Degree of Industrial Engineering\Management

Faculty of Graduate Studies
The University of Jordan

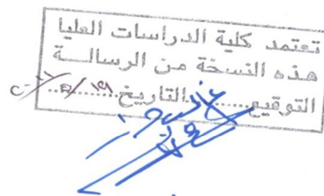


May, 2011

COMMITTEE DECISION

This Thesis/Dissertation (Q, R MODEL FOR DETERIORATING INVENTORY WITH DELAY IN PAYMENT AND BACK ORDER CANCELLATION) was Successfully Defended and Approved on May 9, 2011.

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Dedication

This thesis is dedicated to my father, who taught me that the best kind of knowledge to have is that which is learned for its own sake. It is also dedicated to my mother, who taught me that even the largest task can be accomplished if it is done one step at a time. Also it is dedicated to my Husband Belal, without his patience, understanding, and support the completion of this work would not have been possible.

AKNOWLEDGEMENT

A major research project like this is never the work of anyone alone. The contributions of many different people, in their different ways, have made this possible. I would like to extend my appreciation especially to the following. Thank God for the wisdom and perseverance that he has been bestowed upon me during this research project, and indeed, throughout my life: "I can do everything through him who gave me strength, my parents, to whom I dedicate my work. I am thankful for their love and best wishes, which has helped me in the successful completion of my study, my Husband 'Belal,' for his continual support and encouragement throughout this year. I'm heartily thankful to my supervisor, Dr. Mahmoud Barghash, whose encouragement, guidance and support from the initial to the final level enabled me to develop an understanding of the subject. Lastly, I offer my regards and blessings to all of those who supported me in any respect during the completion of the project.

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LIST OF ABBREVIATIONS OR SYMBOLS

EOQ: Economic Order Quantity

GA: Genetic Algorithm

TAC: total Annual Cost

SKU: Stock Keeping Unit.

LTD: Lead Time Demand

EA: evolutionary algorithms.

Q: Order quantity

R: replenishment point.

D: Annual Demand

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Abstract

Inventory control is important to ensure quality control in businesses that handle transactions revolving around consumer goods. A good inventory control system will alert the retailer when it is time to reorder to avoid stock-outs, to prevent shrinkage (spoilage/theft), and to provide proper accounting. Our study tends to develop a model for constant deteriorating rate inventory under permissible delay in payment and back order cancellation when lead time demand follows a normal distribution, where we extended Wu's (2001) model to include deterioration inventory and partially backlogged backorders.

The minimization of the total variable cost per unit of time is taken as the objective function to find the optimal Order quantity (Q) and reordering point (R). Mathematical model was developed for the inventory problem, then two solution procedures was used to solve the developed model which they are a simple to use algorithm and Genetic algorithm. MATLAB program was used to find the results of the two solution procedures. The model was validated using numerical example used in Wu (2001). Also sensitivity analysis was conducted to study the effect of model parameters on the (Q, R and total annual cost (TAC)). And according to the results of numerical example some recommendations was made to help minimize TAC of inventories.

The results of two solution procedures, simple to use algorithm and Genetic algorithm, were compared and the results shows that the genetic algorithm gives less value for Q and R than the simple-to-use algorithm but the same value of TAC. Sensitivity analysis shows that the optimal order quantity (Q) is moderately sensitive to changes in annual demand (D) and unit purchase cost (p), and optimal reorder point (R) is less sensitive to changes in the D and p. But, the minimum TAC is highly sensitive to changes in the value of D and p. Furthermore, to decrease the TAC and keeping Q and R constant we have to increase the interest earned from sales during credit period. Also Increasing cancellation rate an cost of lost customer good well will increase the TAC largely but deterioration rate has a slight effect on the values TAC.

Chapter 1: Introduction

1.1 Background

Inventory control is important to ensure quality control in businesses that handle transactions revolving around consumer goods. Without proper inventory control, a large retail store may run out of stock on an important item.

Today, inventory management is considered to be an important field in Supply chain management. Once the efficient and effective management of inventory is carried out throughout the supply chain, service provided to the customer ultimately gets enhanced. Hence, to ensure minimal cost for the supply chain, the determination of the level of inventory to be held at various levels in a supply chain is unavoidable. Minimizing the total supply chain cost refers to the reduction of holding and shortage cost in the entire supply chain. Efficient inventory management is a complex process which entails the management of the inventory in the whole supply chain and getting the final solution as an optimal one. In other words, during the process of supply chain management, the stock level at each member of the supply chain should account to minimum total supply chain cost.

There are three kinds of inventory that are of concern to managers:

- Raw materials,
- In-process or semi-finished goods,
- Finished goods.

If a manager effectively controls these three types of inventory, capital can be released that may be tied up in unnecessary inventory, production control can be improved and can protect against obsolescence, deterioration and/or theft. The major reasons for inventory control are:

- To balance the stock as to value, size, color, style, and price line in proportion to demand or sales trends.
- To secure the best rate of stock turnover for each item.
- To reduce expenses and markdowns.
- To maintain a business reputation for always having new, fresh merchandise in wanted sizes and colors.

Inventory control is concerned with minimizing the total cost of inventory and the three main factors in inventory control decision making process are:

- The cost of holding the stock (e.g., based on the interest rate).
- The cost of placing an order (e.g., for row material stocks) or the set-up cost of production.
- The cost of shortage, i.e., what is lost if the stock is insufficient to meet all demand.

Inventory control is not only about storage, as different reports generated with the help of inventory control system will assist management in making many important business decisions. One of the most important inventory control models is the economic order quantity; in conventional economic order quantity (EOQ) model several assumptions must be satisfied, these assumptions include non perishable (deteriorate), no back orders, no discounts, no delay in payment and so on. But in real life in the first hand many products such as vegetables, medicine as well as blood are perishable and may deteriorate with time. In the other hand Suppliers may give their

customers a credit period till next order replenishment, and allow backorders. More over the customer may cancel their back orders before receiving them.

In practice suppliers must stimulate their products due to competitiveness of market, for that they may offer discounts or they may offer delay period to pay for products; during this period "credit period" no interest charged, but if account settlement occurs beyond this period interest must be charged as penalty for being late. When supplier allows fixed credit period for settling the account, it seems that he is giving a loan for customers without interest during credit period. During this period and before the account be settled, the retailer sells his items and accumulate revenue and earn interest instead of paying the over draft which is necessary if supplier wants to be paid immediately when replenishment, for that it makes economic sense for customers to delay the payment up to the last moment of credit period.

Moreover, most of inventories deteriorate due to spoilage, evaporation, damage etc. and deterioration may be continuous with time or constant, deterministic or stochastic, and there are some types of products that may remain fresh for specific period of time and then starts to deteriorate these products called the non-instantaneous deteriorating inventory.

Also in real life shortages are allowed and may be backlogged completely, or partially because back ordered customers lack of patience so they may cancel their backorders before receiving them which affect the total cost for supplier to include lost sales cost and more over it may cost him to lost customer good well.

There are a number of methods used to solve inventory models depending on the nature of each problem; like simple algorithms, dynamic programming, Genetic algorithm (GA) and others. Where GAs are a class of adaptive search techniques based on the principle of population genetics. GA is a search heuristic

that mimics the process of natural evolution. This heuristic is routinely used to generate useful solutions to optimization and search problems. Genetic algorithms belong to the larger class of evolutionary algorithms (EA), which generate solutions to optimization problems using techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover. Our study tends to develop a model for a constant deteriorating rate inventory under permissible delay in payment and back order cancellation when lead time demand follows a normal distribution. And to find the optimal Order quantity (Q) and reordering point(R) we compared the results of two solution procedures which are simple-to-use algorithm and genetic algorithm.

1.2 Objective:

To choose Q and R to minimize the average inventory holding, ordering and backorder costs, deteriorating cost and order cancellation cost (the cost resulted from lost customer good well).

Chapter 2: literature Review

Economic order quantity is the level of inventory that minimizes the total inventory holding costs and ordering costs. It is one of the oldest classical inventory control techniques. The framework used to determine this order quantity is also known as “Wilson EOQ Model” or “Wilson Formula”. EOQ only applies under the following assumptions:

1. The ordering cost is constant.
2. The demand rate is constant
3. The lead time is fixed
4. The purchase price of the item is constant i.e. no discount is offered.
5. The replenishment is made instantaneously; the whole batch is delivered at once.
6. Payment for items is made immediately.

2.1 Inventory systems with delay in payment

In real life these assumptions are not all applicable at the same time, were most of suppliers may offer their customers a credit period to pay for items to stimulate their products market. Goyal (1985) developed a mathematical model to determine the optimum EOQ under permissible delay in payment, he assumes that demand (D) is constant with time, shortages were not allowed and during the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. At the end of this period, the account is settled and paying is started for the interest charges on the items in stock. He derived the formula for EOQ with permissible

delay in payment by finding the first derivative for the total cost function, then he developed an algorithm to find EOQ and the length of replenishment interval that minimizes the cost, Z(T), in two cases; first case when the delay period is less than the length of replenishment cycle, and the other case when delay period is larger than the replenishment cycle. The total annual cost in Goyal (1985) consists of the following elements:

1. Cost of placing an order which is equal to the cost of placing one order (S) divided by the time between two successive orders (T).
2. Cost of stock holding which is DT/2 (see Figure 1) so stock holding per year is DTh/2 where h is cost of holding one unit in stock per year,
3. Cost of interest charges for the items kept in stock; As items are sold, and before the replenishment account is settled, the sales revenue is used to earn interest. When the replenishment account is settled, the situation is reversed, and effectively the items still in stock have to be financed at interest rate I_c, The stock level at the time of settling the replenishment account equals D(T - t) (see Figure 1(a)) and the Interest payable during (T-t) in one cycle is $= Dp(T - t)^2 I_c/2$ and so the interest payable for one year will be $Dp(T - t)^2 I_c/2\,T = \frac{DpTI_c}{2} + \frac{Dpt^2 I_c}{2T} - DptI_c$ where p is unit purchase cost, t is permissible delay in payment and I_c is interest charges per \$ investment in stock per year.
4. Interest earned during the permissible settlement period I_d, The maximum accumulated amount earning interest during the settlement period equals Dtp if $T \geq t$ (see Figure 1(a)) or DTp if $T \leq t$ (see Figure 1(b)), where the dashed line

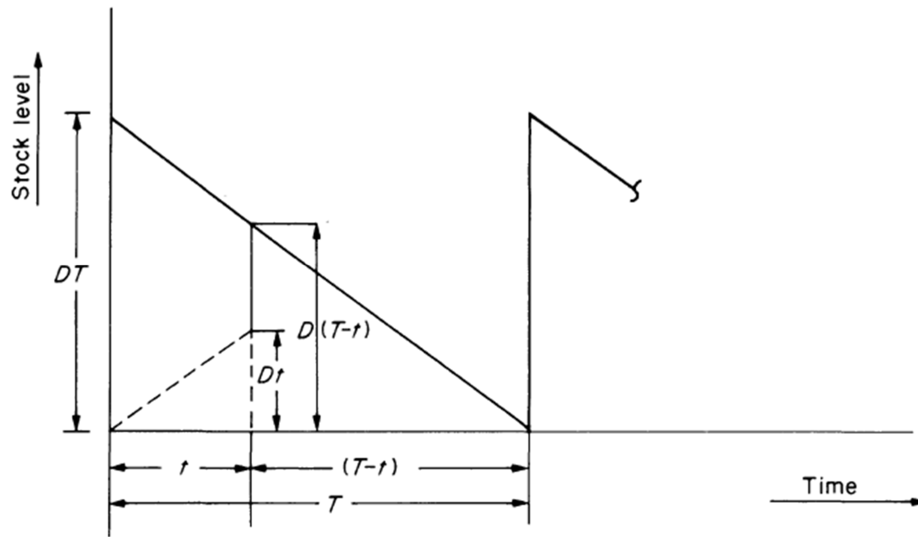
represents the interest earned during settlement period. Hence the interest earned during the permissible settlement period for the two cases is obtained as follows:

Case I: $T \geq t$, interest earned in one cycle is $Dpt^2I_d/2$.

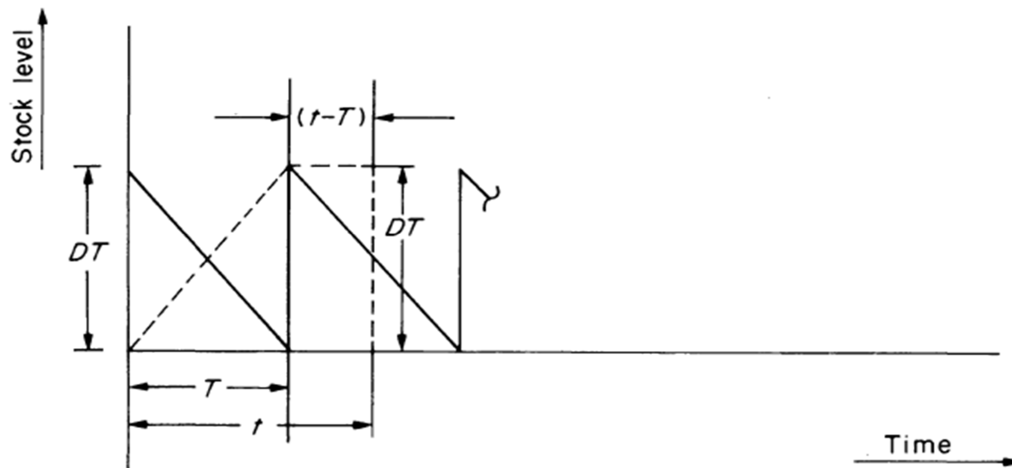
Interest earned in one year is $Dpt^2I_d/2T$

Case II: T ▪ t, interest earned in one cycle is $\left(\frac{DT^2p}{2} + DTp(t-T)\right)I_d = \mathrm{DTpI_d}(t - \frac{T}{2})$.

Interest earned in one year is $\mathrm{DpI_d}(t - \frac{T}{2})$



a) Time weighted inventory during $T \geq t$.



b) Time weighted inventory during $T \leq t$.

Figure 1: (a) and (b) Time weighted inventory during (T-t), Goyal(1985).

For case I, when finding the first derivative of the total annual cost $Z(T) = (2S + Dpt^2(I_c - I_d))/2T + DT(h + pI_C)/2 - DpTI_C$ and equating it to zero the economic order quantity for T= $T_1^* = \sqrt{(2S + Dpt^2(I_c - I_d)/D(h + pI_c)}$ is $Q(T_1^*) = (D(2S + Dpt^2 I_c)/(h + pI_c))^{1/2}$, and for Case II, when finding the first derivative of the total annual cost Z(T)= $S/T + DT(h + pI_d)/2 - DptI_d$ the economic order quantity for T= $T_2^* = \sqrt{2S/D(h + pI_d)}$ is $Q(T_2^*) = (2DS/(h+pI_d))^{1/2}$, where T_1^* and T_2^* are the optimal cycle time for case I and case II respectively. In order to determine the economic operating policy he developed this simple algorithm which is:

Step 1: Determine T_1^* If $T_1 \geq t$, obtain $Z(T_1^*)$ from Case I.

Step 2: Determine T_2^* if $T_2 < t$, evaluate $Z(T_2^*)$ from Case II.

Step 3: If $T_1^* < t$ and $T_2^* \geq t$, then evaluate Z(t) when T=t.

Step 4: Compare $Z(T_1^*)$, $Z(T_2^*)$ and Z(t). Select the replenishment interval and the order quantity associated with the least annual cost value evaluated in steps 1 and 2 or 3. The results of his model shows that when supplier offers credit period the order quantity and the replenishment interval will increase considerably and total cost will decrease compared with traditional EOQ model values, Chung and Huang(2009) extended Goyal's generalized model by allowing shortages in Goyal model. Where the optimization problem in their paper have two variables i.e. the length of period with positive inventory and length of period with negative inventory as compared to Goyal's model which dealt with one decision variable, i.e. the length of replenishment cycle. A proof is given that the total annual variable cost function of this new inventory model possesses is convex. In their study Chung and Huang (2009) follows Goyal methodology to find order quantity and more over; they prove the convexity of the cost function. They found that their model reduces to that with no allowable shortage when

the shortage cost is extremely high. It is also noted that the total annual variable cost function is convex. But Chung (1998) follows another approach to find the EOQ under conditions of permissible delay in payment, where he used the same assumption that have been used by Goyal (1985) but he first proved the convexity of cost function, then he developed a simple procedure to find optimal cycle length that minimizes cost which is less elaborate than Goyal (1958) procedure. He Concluded based on his results that the order quantity with delay in payment is larger than the classical EOQ.

Sana and Chaudhuri (2008) studied various types of deterministic demand when delay in payment is permissible also they included supplier offers of percent discount on purchased items, their study developed based on the idea that the retailers have to correlate between the selling price and supplier's trade offer keeping in mind profit maximization strategy. In their model all increasing deterministic demands are discussed in the environment of permissible delay in payment and discount offer to the retailer. They assumed that the inventory system involves a single item type, shortages are not permitted, replenishment is instantaneous, Lead time is neglected, permissible delay in payment to the supplier by the retailer is considered, the supplier offers different discount rates of price at different delay periods and planning horizon is infinite. Sana and Chaudhuri (2008) developed theorems to calculate the optimum cycle time and order quantity for the following scenarios:

A. **when demand rate is dependent on inventory level:** In this case, the demand rate is endogenous to the retailer and is a function of inventory level The effect of this dependency is that the retailer has incentive to keep higher levels of inventory despite higher holding costs as long as the item is profitable and the demand is an increasing function of the inventory level. This, in turn, results in additional sales, higher replenishment rates and gear up potential profits

B. When demand rate is dependent on ordered quantity: This trend in demand applies to some established products (like essential commodities, seasonal goods, etc.) having a demand rate steadily increasing or accelerating over time with the increase of customers.

C. When demand rate is linearly varying with time

D. When demand is constant.

E. When demand is exponentially varying with time In the case of some products (e.g., new computer chips, seasonal goods, etc.), the demand rate is likely to increase very fast, almost exponentially, with time while in the case of some other products (e.g., spare parts of obsolete machines, off-season goods, etc.), the demand rate is expected to decrease very fast, almost exponentially with time.

F. When demand rate is dependent on price: In this case, demand is a function of price (p), where p is a decision variable.

G. When demand rate is dependent on price stock

When applying their model on a numerical example they found that:

For scenario A: As the average profit function is highly nonlinear, a closed type formula for concavity of the function is not required, but the function is concave. Hence the required better optimal solution is a global maximum.

For scenario B: As the average profit function is highly nonlinear, a closed type formula for concavity of the function is not required, but the function is concave. Hence the required better optimal solution is a global maximum.

For scenario C: As the average profit function is highly nonlinear, a closed type formula for concavity of the function is not required, but the function is concave. Hence the required better optimal solution is a global maximum.

For scenario D: As the average profit function is nonlinear, a closed type formula for concavity of the function is not required, but the function is concave. Hence the required better optimal solution is a global maximum.

For scenario E: As the average profit function is nonlinear, a closed type formula for concavity of the function is not required here, but the function is concave. Hence the required better optimal solution is a global maximum.

For scenario F: As the average profit function is nonlinear, a closed type formula for concavity for the function is not required, but the function is concave. Hence the required better optimal solution is a global maximum.

And for scenario G: As the average profit function is nonlinear, a closed type formula for concavity of the function is not required, but the function is concave. Hence the required better optimal solution is a global maximum.

Delay in payment in real life practice must be subject to convention i.e. the delayed payment must be related to threshold quantity if buyer ordered less than threshold, he must pay immediately, Chung, et al. (2005) Deals with this problem of EOQ with permissible delay in payment taking into account the ordered quantity, that is when ordered quantity is less than threshold quantity at which delay in payment is permitted then payments for products must be made immediate. He assumed that demand rate (D) is known and constant, shortages are not allowed, time period is infinite and replenishments are instantaneous with a known and constant lead time. When the retailer must pay the amount of purchasing cost to the supplier, the retailer will borrow 100% purchasing cost from the bank to pay off the account with rate when cycle time (T) is greater than or equal to credit period (M); the retailer returns money to the bank at the end of the inventory cycle. However, when, T, is less than or equal to M; the retailer returns money to the bank at T = M. Finally if the credit period is shorter

than the cycle length, the retailer can sell the items, accumulate sales revenue and earn interest with rate throughout the inventory cycle. The main issue discussed in their paper that If order quantity is less than or equal to a fixed quantity, W, the delay in payments is not permitted. Otherwise, the fixed trade credit period M is permitted. They developed e the solution procedure to find best cycle time for tow cases as follows:

(i) M≥W/D:

This case has three scenarios when:

Scenario 1: 0 T≤W/D, (see Figure 2).

The total annual variable cost is TVC_1 (T) $= A/T + DTh/2 + cI_p DT - DTsI_e/2$

By equating the first derivative of the annual variable cost to zero the optimal cycle time for this scenario is $T_1^* = (2A/D(h+2cI_e-sI_e))^{1/2}$

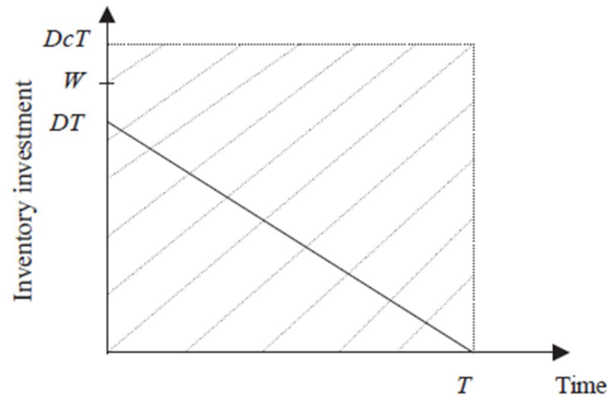


Figure 2: Total accumulation of interest payable when0 T≤W/D, Chung, et al. (2005).

Scenario 2: W/D≤T ≤M

The total annual variable cost is TVC_2 (T) $= A/T + DTh/2 + cI_p DT - DTsI_e/2$

By equating the first derivative of the annual variable cost to zero the optimal cycle time for this scenario is $T_2^* = (2A/D(h+sI_e))^{1/2}$

And Scenario 3: M≤T, (see Figure 3).

The total annual variable cost is $TVC_3(T) = A/T + DTh/2 + cI_p D(T-M) - DTsI_e/2$

By equating the first derivative of the annual variable cost to zero the optimal cycle time for this scenario is, where T_1^*, T_2^* and T_3^* is the optimal value of cycle time for each secenario1,2 and 3.

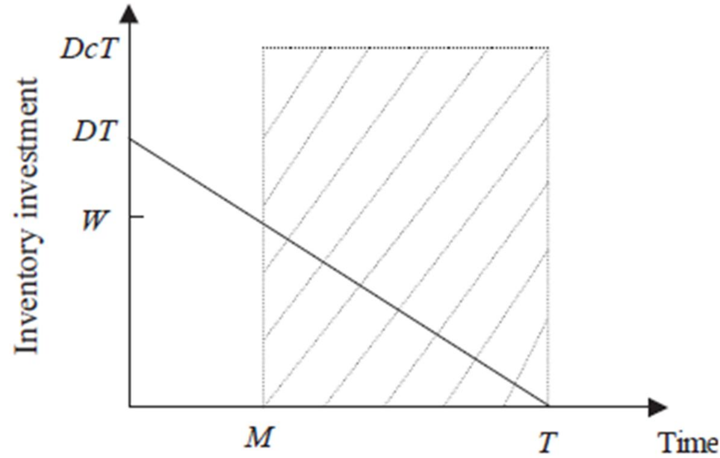


Figure 3: Total accumulation of interest payable when M≤T, Chung, et al. (2005).

(ii) M ▪ W/D:

The annual variable cost TVC (T)= $A/T + DTh/2 + cI_p DT - DTsI_e/2$

For this case the optimal cycle time T^*= W/D

where Q is the order quantity, A is the cost of placing one order, c is the unit purchasing price per item, s is the unit selling price per item, h is the unit stock holding cost per item per year excluding interest charges, I_e is the interest rate that can be earned per $ per year, I_p is the interest rate charged per $ investment in inventory per year, T* the optimal cycle time of TVC (T).To solve the above developed model Chung, et al. (2005) developed an algorithm to help to decide optimum cycle time. Furthermore, they developed theorems to reveal the solution procedure to find optimum cycle time when W = 0.Finally, they develops some comparisons with Goyal's model (1985) and demonstrate that the optimal cycle time is not longer than that of Goyal's model (1985).

Huang (2007) developed an inventory model where supplier offers retailer partial credit period when the ordered quantity is less than predetermined one. Under this condition, he modeled the retailer's inventory system as a cost minimization problem to determine the retailer's optimal inventory cycle time, T, and optimal order quantity, Q. Three theorems are established to describe the optimal replenishment policy for the retailer. He assumes that replenishments are instantaneous, demand rate, D, is known and constant, shortages are not allowed, the inventory system involves only one type of inventory and time horizon is infinite. If $Q < W$, i.e. $T < W/D$, where W is quantity at which the fully delay payments permitted per order the partially delayed payment is permitted. Otherwise, fully delayed payment is permitted. Hence, if $Q < W$, pay cQ where c is unit purchasing price, after M time periods from the time the order is filled. Otherwise, as the order is filled, the retailer must make a partial payment (1-α)cDT, where α is the fraction of the delay payments permitted by the supplier per order and $0 \leq \alpha \leq 1$, to the supplier, then the retailer must pay off the remaining balances, αcDT, at the end of the trade credit period. And During the time period that the account is not settled, generated sales revenue is deposited in an interest bearing account. Their model for the three mentioned cases is as follows:

Case I: where $M \geq W/D$, this case has three sub cases:

(i) $M \leq T$, the optimal cycle time for this sub case is $T^* = T_1^* = \sqrt{(2A + cDM^2(I_k - I_e)/D(h + cI_k))}$ and the annual total relevant cost $TRC_1(T) = A/T + DTh/2 + cI_k D(T-M)^2/2T - DM^2 cI_e/2T$

(ii) $W/D \leq T \leq M$, the optimal cycle time for this sub case is $T^* = T_2^* = \sqrt{(2A/D(h + cI_e))}$

And the annual total relevant cost $TRC_2(T) = A/T + DTh/2 - cI_e DT(M - T/2)/T$

(iii) 0・ T・ W/D, the optimal cycle time for this sub case is

$T^* = T_3^* = \sqrt{(2A/(D(h+c[(1\text{-}\alpha)2I_k + I_e]))}$,and the annual total relevant cost is:

$TRC_3(T) = A/T + DTh/2 + (1-\alpha)^2 cI_k DT^2/2T - cI_e DT(M - T/2)/T$.(see Figure4):

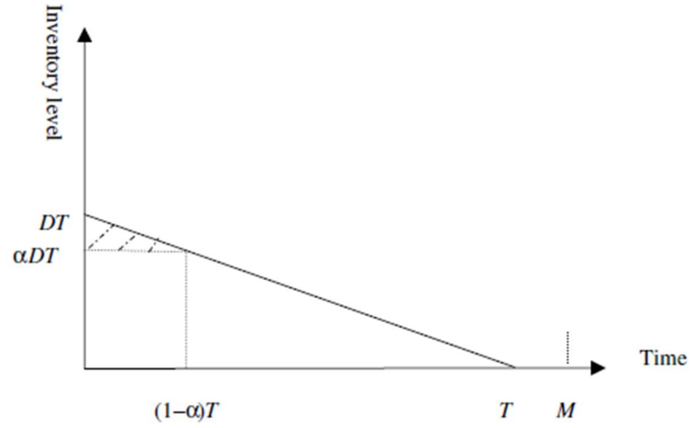


Figure 4: The inventory level and the total saved amount of interest payable when 0・ T・ W/D for Huang (2007).

Case II: when M・ W/D≤M/(1-α), for this the annual total relevant cost is

$TRC_4(T) = A/T + DTh/2 + cI_k D[(1\text{-}\alpha)^2 T^2 + (T\text{-}M)^2]/2T - cI_e DM^2/2T$, (see Figure 5),

and the optimal cycle time is $T_4^* = \sqrt{(2A + cDM^2(I_k - I_e)/(D(h + cI_k[1 + (1\text{-}\alpha)^2]))}$

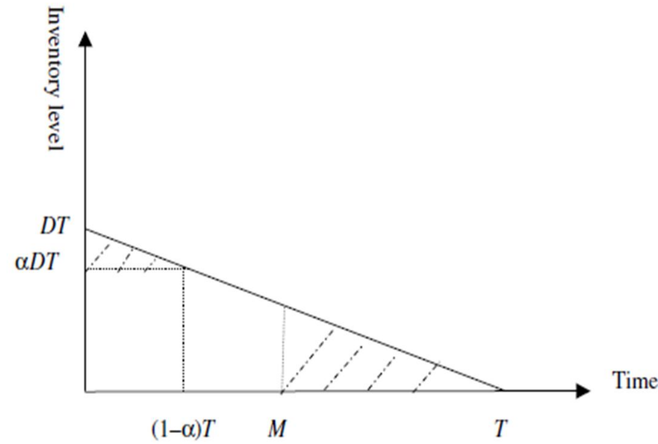


Figure 5: The inventory level and the total saved amount of interest payable when M・ W/D≤M/(1-α), Huang (2007).

Case III: when $M/(1-\alpha)$▪ W/D,the total annual relevant cost is $TRC_5(T) = A/T + DTh/2 + cI_k DT(T/2 - \alpha M)/T - cI_e DM^2/2T$, (See Figure 6), and the optimal cycle time for this case is $T_5^* = \sqrt{(2A - cDM^2 I_e/(D(h + cI_k))}$

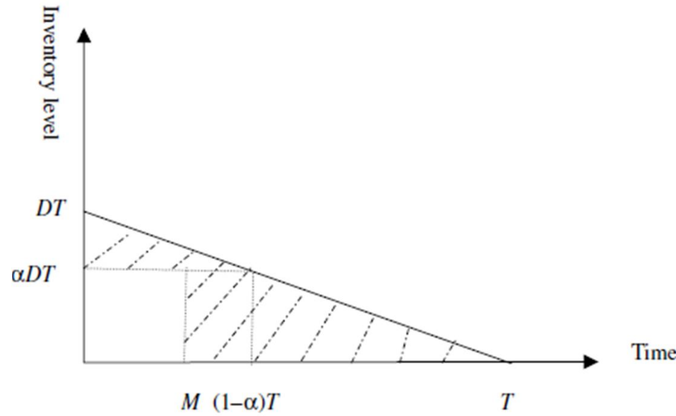


Figure 6: The inventory level and the total saved amount of interest payable when $M/(1-\alpha)$▪ W/D, Huang (2007).

Huang (2007) in his above mentioned model come out with the following results; a higher value of the fraction of the delay payments permitted brings about a larger order quantity and smaller annual total relevant costs; a higher value of the minimum order quantity as required to obtain fully permissible delay in payments brings about a smaller order quantity and larger annual total relevant costs; and a higher value of unit purchasing price brings about a smaller order quantity and smaller annual total relevant costs. From the viewpoint of supplier's marketing policy, the supplier can use the fraction of the delay payments permitted to control more agilely the effects of stimulating the demand from the retailer. For example, the supplier can offer the larger fraction of the delay payments permitted to stimulate the larger order quantity from the retailer. On the other hand, the supplier can use the smaller fraction of the delay payments permitted to decrease the order quantity of the retailer. As such, the more realistic and flexible marketing policy is valuable to the supplier.

Teng, et al.(2005) complement the assumption that the selling price is the same as purchasing cost and developed an algorithm that optimize the price and order quantity when delay in payment is permissible. They developed an algorithm for a retailer to determine its optimal price and lot size simultaneously when the supplier offers a permissible delay in payments. They assumed for their model that the demand for the item is a downward sloping function of the price, for simplicity, they assumed that demand is a constant elasticity function of the price, Shortages are not allowed. In reality, the retailer has numerous ways to spend the profit from sales, such as expansion, new product development, hardware and software upgrade, etc. For simplicity, they assumed here that the retailer spends the profit in other activities than paying off the loan. During the time the account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of this period, the retailer pays off all units sold, keeps the rest for the use of the other activities, and starts paying for the interest charges on the items in stocks. In the meantime, the retailer starts accumulating profit for the use of the other activities and time horizon is infinite. They established the necessary and sufficient conditions for the unique optimal replenishment interval and their model developed for two cases:

Case I: when $T \leq m$, (see Figure 7). As a result, the total annual profit

$$Z(T,p) = Z_1(T,p) = pD - s/T - D(h + c\theta)(e^{\theta T} - 1)/\theta^2 T + hD/\theta + pI_d D(m - T/2)$$

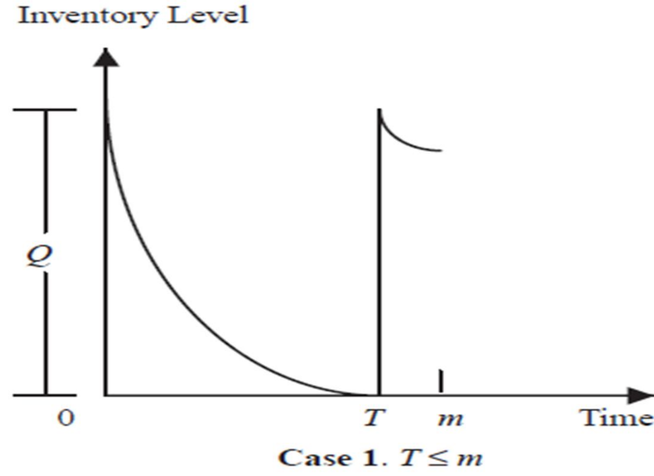


Figure 7: Graphical representation of inventory system for Case 1, Teng, et al.(2005).

By taking the first derivative for Z_1(T,p), the total annual profit function for case I, with respect to T and equating it to zero the optimal replenishment time will be:

$$T = T_1 = \sqrt{2S/D(h + c\theta + pI_d)}$$

And the optimal ordering quantity Q^*(T) will be

$$Q^*(T) = Q^*(T_1) = \sqrt{2sD/(h + c\theta + pI_d)} + \theta s/(h + c\theta + pI_d)$$

CaseII: when T≥m, (see Figure 8). The total annual profit is

$$Z(T, p) = Z_2(T, p) = pD - s/T - D(h + c\theta)(e^{\theta T} - 1)/\theta^2 T + hD/\theta + cI_c D(e^{\theta(T-m)} - 1)/\theta^2 T + cI_c D(T - m)/\theta T + pI_d Dm^2/2T$$

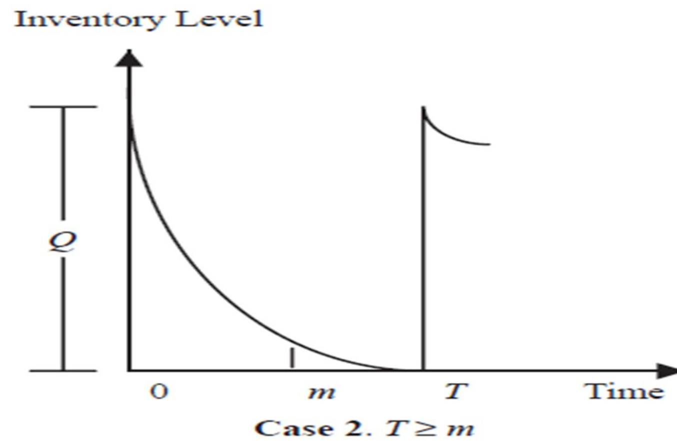


Figure 8: Graphical representation of inventory system for Case 2 in Teng, et al.(2005).

Also here when taking the first derivative for $Z_2(T,p)$, the total annual profit function for case II, with respect to T and equating it to zero the optimal replenishment time will be:

$$T = T_2 = \sqrt{[2s + Dm^2 9cI_c - pI_d)]/[D(h + c\theta + cI_c)]}$$

And the optimal order quantity for this case is:

$$Q^*(T) = Q^*(T_2) = \sqrt{[2sD + D^2m^2(cI_c - pI_d)]/[h + c\theta + cI_c]}$$
$$+\theta[2s + Dm^2(cI_c - pI_d)]/\{2[h + c\theta + cI_c)\}$$

Where, h is the unit holding cost per year excluding interest charges, p is the selling price per unit, c is the unit purchasing cost, with c ▪ p, I_c is the interest charged per $ in stocks per year by the supplier, I_d is the interest earned per $ per year, s is the ordering cost per order, m is the period of permissible delay in settling account; that is, the trade credit period. Q is the order quantity, θ is the constant deterioration rate, where 0≤θ ▪ ▪ 1, I(t) the level of inventory at time t where $0 \le t \le T$, T is the replenishment time interval, and D is the annual demand, as a decreasing function of price; where $D(p)=\alpha p^{-\beta}$ where $\alpha>0$ and $\beta>1$ and Z(T,p) is the annual profit. T_1 and T_2 are optimal replenishment time for case I and case II respectively and $Q^*(T_1)$,$Q^*(T_2)$ are optimal order quantities for case I and case II respectively.

they developed an algorithm to find the optimal solution they then compare the classical EOQ with the proposed model, where they found that the classical EOQ value is higher than the optimal order quantities results from the developed model, also they found that a higher value of the permissible delay causes a higher value of the unit profit but lower values of the selling price and the replenishment cycle time.

2.2 inventory systems for deteriorating items with delay in payment

Ouyang, et al. (2009) developed generalized mathematical model by relaxing the classical EOQ model in the following ways:

1) Retailers selling price is higher than its purchase cost
2) Interest rate is not necessary higher than the retailers investment return rate
3) Many selling items deteriorate continuously and
4) Supplier may offer partial delay in payment if order quantity is less than pre-determined one.

In their study they generalized Goyal (1985) model. Then they established the proper mathematical model, where:

D: the annual demand

A: the ordering cost per order

W: the quantity at which the fully delay payment permitted per order

c; the purchasing cost per unit

h: the unit holding cost per year excluding interest charge

p: the selling price per unit

I_e : the interest earned per dollar per year

I_k : the interest charged per dollar in stocks per year

M: the period of permissible delay in settling accounts

α A: the fraction of the delay payments permitted by the supplier per order, $0 \leq \alpha \leq 1$

θ; the deterioration rate, 0 ▪ h < 1

T: the replenishment cycle time in years

Q; the order quantity

TRC (T): the annual total relevant cost, which is a function of T

T*: the optimal replenishment cycle time of TRC(T)

Q*: the optimal order quantity their established mathematical model is as following:

Case 1: $T_0 > M \geq T_w$, TRC(T)=
$$\begin{cases} TRC_1(T), & M \leq T, \\ TRC_2(T), & Tw \leq T \leq M \\ TRC_3(T), & 0 ▪ T \leq Tw \end{cases}$$

Case 2: $T_0 \geq Tw > M$, TRC(T)=
$$\begin{cases} TRC_1(T), & Tw \leq T, \\ TRC_4(T), & M \leq T ▪ Tw \\ TRC_3(T), & T \leq M \end{cases}$$

Case 3: T>T0>M, TRC(T)=
$$\begin{cases} TRC_1(T), & Tw \leq T, \\ TRC_5(T), & T_0 \leq T ▪ Tw \\ TRC_4(T), & M \leq T \leq T0 \\ TRC_3(T), & T \leq M \end{cases}$$

Where,

$$TRC_1(T) = A/T + (c\theta + h)(e^{\theta T} - \theta T - 1) D/\theta^2 T + cIkD [e^{\theta(T-M)} - \theta(T - M) - 1]/\theta^2 T - pI_e DM^2/2T$$
$$TRC_2(T) = A/T + (c\theta + h)(e^{\theta T} - \theta T - 1) D/\theta^2 T - pI_e D (M - T/2)$$

$$\begin{aligned} TRC_3(T) = {} & A/T + (c\theta + h)(e^{\theta T} - \theta T - 1)D/\theta^2 T + cI_k (c/p)(1-\theta)^2 D(e^{\theta T} - 1)^2/2\theta^2 T \\ & - pI_e D[T - (1-\theta)(c/p)(e^{\theta T} - 1)/\theta]^2 - pI_e D(M - T)[T - (1-\theta)(c/p)(e^{\theta T} - 1)/\theta] \end{aligned}$$

$$\begin{aligned} TRC_4(T) = {} & A/T + (c\theta + h)(e^{\theta T} - \theta T - 1)D/\theta^2 T + cI_k (c/p)(1-\theta)^2 D(e^{\theta T} - 1)^2/2\theta^2 T \\ & + cI_k D[e^{\theta(T-M)} - \theta(T - M) - 1]/\theta^2 T - pI_e D[M - (1-\theta)(c/p)(e^{\theta T} - 1)/\theta]^2 \end{aligned}$$
$$\begin{aligned} TRC_5(T) = {} & A/T + (c\theta + h)(e^{\theta T} - \theta T - 1)D/\theta^2 T + cI_k (c/p)(1-\theta)^2 D(e^{\theta T} - 1)^2/2\theta^2 T \\ & + cI_k \theta D(e^{\theta T} - 1)[(1-\theta)(c/p)^{\theta} (e^{\theta T} - 1)/\theta - M] \end{aligned}$$

Then they derived several theoretical results to determine the optimal solution under various situations and used two approaches to solve this complex inventory problem. The first approach is to use any standard nonlinear programming software to solve 10 sub-cases in which the objective function is non-linear and the

second approach is to develop an algorithm to solve the inventory problem. The assumption used by Ouyang, et al. (2009) was the same as the ones used in Huang (2007). Based on their developed model; if the retailer's optimal order quantity is less than W (i.e., the partially delayed payment is permitted) and the fraction of the delay payments permitted α is increasing, then the optimal replenishment cycle time T^* and order quantity Q^* will be increasing while the optimal annual total relevant cost TRC (T^*) will be decreasing. When the quantity at which the fully delayed payment is permitted per order W increases, the retailer should take the partially delayed payment (i.e., the optimal order quantity $Q^* < W$) instead of the fully delayed payment (i.e.$Q^* \geq W$), but when the purchasing cost per unit c increases, then the optimal replenishment cycle time T^* and order quantity Q^*will be decreasing while the optimal annual total relevant cost TRC (T^*) will be increasing. However, for the case with α = 0.2 and W=150 in the numerical example, the optimal replenishment cycle and order quantity are fixed and are not affected by the increase in the unit purchase price. The reason is that in this situation, the retailer trades off the benefits of full delay in payment against the partial delay in payment and always enjoys the full delay in payment.

Some researchers studied the delay in payment phenomenon for deteriorating items, because most of inventory items in real life are subject to deterioration in some way. Liao, et al. (2000) developed inventory model for deteriorating items with delay in payments considering inflation rate when shortages were not allowed and consumption rate depend on initial stock. The effect of the inflation rate, deterioration rate, initial-stock-dependent consumption rate and delay in payment are discussed. In their study, mathematical models are also derived under two different circumstances, i.e., **Case I:** The credit period is less than or equal to the cycle time for settling the account; and **Case II:** The credit period is greater than the cycle

time for settling the account. Besides, expressions for an inventory system's total cost are derived for these two cases. Moreover, a computational procedure and GINO were proposed to obtain the optimal order size and cycle time. The results can help managers determine the optimal total cost where the following assumptions were made:

a) The unit price is subject to the same inflation rate as other inventory related costs, thereby implying that the ordering size can be determined by minimizing the total cost over a planning period.
b) The inflation rate is constant.
c) The replenishment rate is infinite, i.e., the replenishment is instantaneous.
d) Backlogging is not allowed.
e) Lead time is zero.
f) The demand rate is known and constant.
g) Initial-stock-dependent consumption rate is assumed, in which the demand rate depends on the order size follows the function $\lambda = \alpha + \beta Q^{\tau}$, where α, β, τ are positive constants and $Q \neq 0$.
h) The inventory carrying charge is a constant.
i) There is no repair or replenishment of the deteriorated inventory during a cycle time.
j) During the fixed credit period *M*, a deposit is made of the unit cost of generated sales revenue into an interest bearing account. The daily expenses of the system can be met by retaining the difference between retail price and unit cost. At the end of the credit period, the account is settled and interest charges are payable on the account in stock.

And they used the following notations: *H is* length of planning horizon, T is the cycle time, *I* is the inventory level*,* I_t is the inventory level at time *t,* *Q* is the order size, *k* is

the constant rate of inflation ($/$/unit time), $C(t)$ is the unit purchase cost for an item bought at Time t, that is, $C(t) = C_0 e^{kt}$ where C_0 is the unit price at time zero, $A(t)$ is the ordering cost for an order placed at time t, that is $A(t)=A_0 e^{kt}$, where A_0 is the ordering cost at time zero, I_i is the inventory holding cost per unit per year excluding interest charges, θ is the a constant fraction of the on-hand inventory which deteriorates per unit time, I_e is the annual interest that can be earned per unit, I_c is the annual interest charges payable per unit (Note: We generally have $I_c > I_e$), M is the permissible delay period for settling accounts, Q^*_1 is the optimal order size in Case I, Q^*_2 the optimal order size in Case II, T^*_1 the optimal cycle time in Case I, T^*_2 the optimal cycle time in Case II and TC (H,T) is the total system cost over (0,H).

As mentioned above they developed their mode for two cases as following:

Case I: M≤T

$$TC(H,T)=TC_1(H,T) = C_r+C_p+C_c+C_i-C_{e1}$$

Case II: M>T

$$TC(H,T)=TC_2(H,T)= = C_r+C_p+C_c+-C_{e2}$$

Where,

$$\text{replenishment cost} C_r = A_0 (e^{kH} - 1/e^{kT} - 1)$$

$$\text{purchasing cost } C_p = QC_0 (e^{kH} - 1/e^{kT} - 1)$$

$$\text{holding cost } C_c = \lambda I_i C_0 (e^{\theta T} - \theta T - 1)(e^{kH} - 1/e^{kT} - 1)$$

interest charged for inventory not bing sold after (M)

$$C_i = \lambda I_c C_0 [(M - T) + 1/\theta (e^{\theta (T-M)} - 1)(e^{kH} - 1/e^{kT} - 1)]/\theta$$

$$\text{interest earned for case (I) } C_{e1} = \lambda I_e C_0 M^2 (e^{kH} - 1/e^{kT} - 1)/\theta^2$$

$$\text{interest earned for case (II)} C_{e2} = \lambda I_e C_0 \theta (TM - 1/2T^2)(e^{kH} - 1/e^{kT} - 1)$$

$$\text{And } T = N(Q) = -1 + \sqrt{1 + (2Q\theta/(\alpha + \beta Q\tau))/\theta}$$

To solve their model they used the commercial software GINO, and the total cost and cycle time in their model can be obtained as fo$_2$llowing:

Step 1: Determine Q^*_1 from TC_1 and T^*_1 from N(Q).

Step 2: Determine Q^*_2 from TC_2 and T^*_2 from N(Q), if T^*_2 ▪ M, obtain TC (H, Q^*_1).

Step 3: by comparing TC_1 (H, Q^*_1) and TC_2(H, Q^*_2) select the order size and cycle time with the least total cost evaluated in step 1,step2. Numerical results indicate that an increase in the permissible delay causes both the order size and cycle time to increase. Moreover, the total cost is markedly reduced. The intuitive reason is that, when the permissible payment period increases, the purchaser earns more by investing the cash from the sales of inventory resulting in lower costs. Also, the purchaser tends to hold more inventories which extends the cycle time. The results of sensitivity analysis indicated that the delay period and *inflation rate* are more sensitive toward the optimal total cost. That is, for the delay period, *inflation rate* and deterioration rate, the effects of delay in payment and inflation are strong. Ouyang, et al.(2006) attempts to develop inventory model for non-instantaneous deteriorating inventory with permissible delay in payment, in their model shortages were not allowed ,demand considered constant over time and deterioration rate follows exponential distribution. Chung (2009) proves Ouyang, et al. (2006) theorems. In their model Ouyang, et al. (2006) assumes that the annual demand rate for the item, D, is constant. Replenishment rate is infinite. Shortages are not allowed. The product life (time to deterioration) t has a probability density function $f(t) = \theta e^{-\theta\,(t-td)}$ for t > td, where td is the length of time in which the product has no deterioration and θ is a parameter, t_d is a given constant. During the trade credit period, M, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the period, the retailer pays off all units bought, and starts to pay the capital opportunity cost for the items in stock. Time horizon is

infinite. T is the length of replenishment cycle. A, h, p, c, I_c and I_e denote the ordering cost per order, the holding cost per unit per year excluding interest charges, the selling price per unit, the purchasing cost per unit, the capital opportunity cost in stock per dollar per year and the interest earned per dollar per year, respectively. All of the parameters are positive. $Z_i(T)$ is the total annual relevant inventory cost for i = 1 and2 where i is case number. T^*_i is the optimal length of the replenishment cycle for $Z_i(T)$,respectively. Q^* is the optimal value of order quantity and Z^* is the optimal value of annual relevant cost. And in their developed model they considered two cases as below:

<u>**Case 1:**</u> **M≤td**, (see Figure 9):

$$Z_1(T) = \begin{cases} Z_{11}(T) & \text{if } 0 ▪ T \le M \\ Z_{12}(T) & \text{if } M \le T \le td \\ Z_{13}(T) & \text{if } td \le T \end{cases}$$

Where,

$$Z_{11}(T) = A/T + hDT/2 + pI_eD(M-T/2)$$

$$Z_{12}(T)= A/T + hDT/2 + cI_cD(T-M)^2/2T - pI_eDM^2/2T$$

$$Z_{13}(T)= A/T + D[ht_d+cI_c(t_d-M)][e^{\theta(T-td)}-1]/\theta T + d(h+c\theta+cI_c)[e^{\theta(T-td)}-\theta(T-t_d)-1]/\theta^2 T$$
$$- [cI_cD(t_d-M)^2+hDt_d^2]/2T-pI_eDM^2/2T$$

By find the first derivative for $Z_1(T)$ and equating it to zero the optimal cycle time T^* will be:

A. $T^*_1 = T_{11} = \sqrt{2A/D(h + pI_e)}$, if $0 ▪ T_{11} \le M$

B. $T^*_1 = T_{12} = \sqrt{2A - DM^2(pI_e - cI_c)/D(h + cI_c)}$, if $M \le T_{12} \le t_d$

C. $T^*_1 = T_{13} = T_{11}$, if $t_d \le T_{13}$

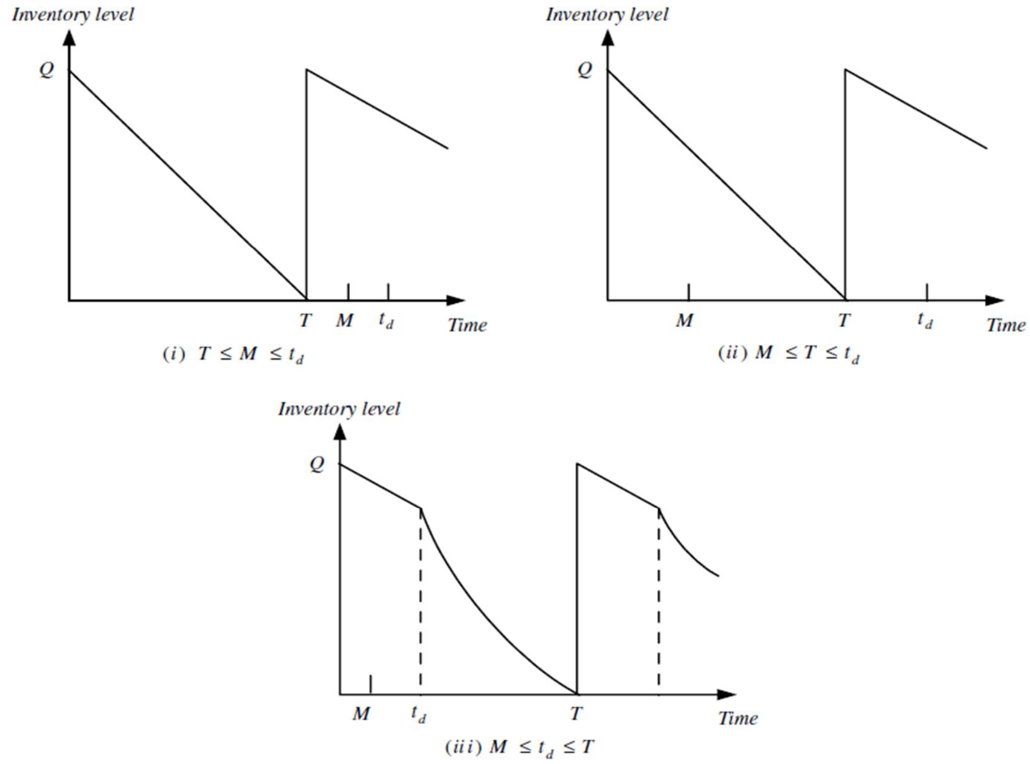


Figure 9: Graphical representation of inventory system for the case **M≤td ,** Ouyang,etal.(2006)**.**

<u>Case 2:</u> M≥td, (see Figure 10):

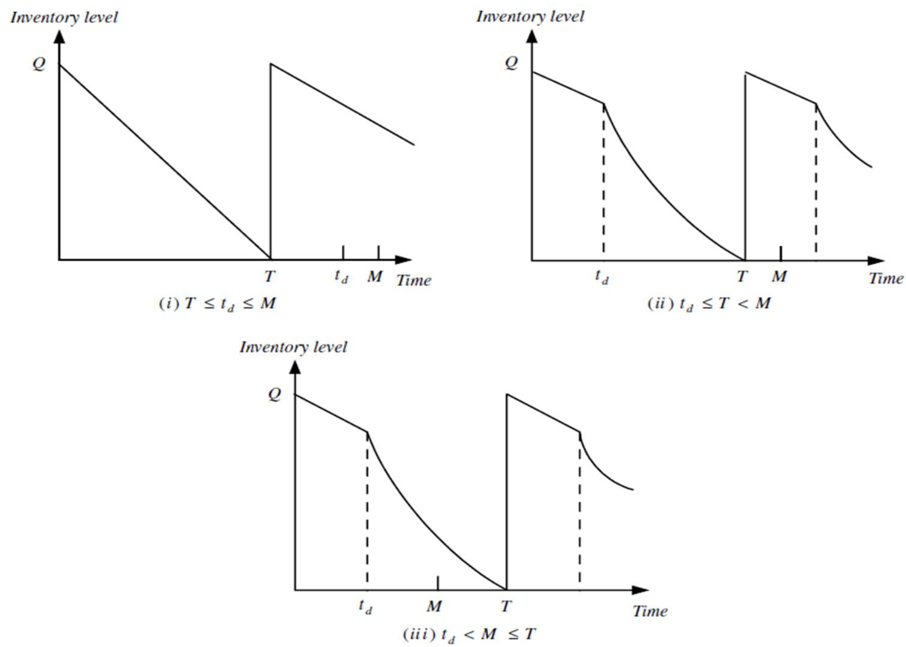


Figure 10: Graphical representation of inventory system for the case **M>td** in Ouyang,etal.(2006)

$$Z_2(T) = \begin{cases} Z_{21}(T) & \text{if } 0 \leq T \leq t_d \\ Z_{22}(T) & \text{if } t_d \leq T \leq M \\ Z_{23}(T) & \text{if } M \leq T \end{cases}$$

Where,

$Z_{21}(T)= A/T + hDT/2-pI_eD(M-T/2)$

$Z_{22}(T)= A/T +hDt_d[e^{\theta(T-td)}-1]/\theta T+ D(h+c\theta)[[e^{\theta(T-td)}-\theta(T-t_d)-1]/\theta^2T+hDt_d2/2T-pI_eDM^2/2T$

$Z_{23}(T)=A/T+hDt_d[e^{\theta(T-td)}-1]/\theta T+D(h+c\theta) [e^{\theta(T-td)}- \theta(t-t_d)-1]/\theta^2T$

$+cI_cD [e^{\theta(T-M)}-\theta(TM)-1]/\theta^2T + hDt_d2/2T+pI_eDM^2/2T$

By find the first derivative for $Z_2(T)$ and equating it to zero the optimal cycle time T^* will be:

D. $T^*_2 = T_{21} = \sqrt{2A/D(h+pI_e)}$, if $0 \leq T_{21} \leq t_d$

E. $T^*_2 = T_{22}$ is difficult to be found, if $t_d \leq T \leq M$

F. $T^*_2 = T_{23}$ is difficult to be found, if $M \leq T$

Based on the above model Ouyang, et al. (2006) concluded that each of T^*, Q^* and Z^* decreases with an increase in the credit period, M, for fixed other parameters. It implies that the longer the credit period supplier offer is, the shorter the replenishment cycle, the lower the order quantity and the total annual relevant inventory cost will be. From economical viewpoint, if the supplier provides a permissible delay in payments, the retailer will order lower quantity in order to take the benefits of the permissible delay more frequently. When the length of time the product has no deterioration t_d increases, and other parameters remain unchanged, the optimal total annual relevant inventory cost Z^* decreases. It implies that the model with non-instantaneous deteriorating items always has smaller total annual relevant inventory cost than with instantaneous deterioration items. Namely, traditional EOQ for instantaneous deteriorating items may likely overvalue the total annual relevant inventory cost. Moreover, if the retailer can

extend effectively the length of time the product has no deterioration for a few days or months, the total annual relevant inventory cost will be reduced obviously. Also when the parameter, h, increases and other parameters remain unchanged, both T^* and Q^* decrease while Z^* increases. Hence if the retailer can reduce effectively the deteriorating rate of item by improving equipment of storehouse, the total annual relevant inventory cost will be lowered.

Also Wu, et al. (2006) considered an appropriate inventory model for non-instantaneous deteriorating items with permissible delay in payment. The purpose of their study is to find an optimal replenishment policy for minimizing the total relevant inventory cost. They developed some useful theorems to characterize the optimal solutions and provide an easy-to-use method to find the optimal replenishment cycle time and order quantity under various circumstances. The non instantaneous deterioration relate to that most goods would have a span of maintaining quality or original condition, namely, during that period, there is no deterioration occurring. They assume that the annual demand rate for the item is constant. Replenishment rate is infinite. Shortages are not allowed. The product life (time to deterioration) t has a probability density function. During the trade credit period, M, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the period, the retailer pays off all units bought, and starts to pay the capital opportunity cost for the items in stock (see Figure 11). They developed some theorems to characterize the optimal solutions and provide an easy-to-use method to find the optimal replenishment cycle time under various circumstances. From the theoretical results, it can be found that the ordering cost is an influential factor when determining an optimal replenishment policy in this study. Furthermore, the higher the ordering cost is, the longer the replenishment cycle time and the greater the order quantity will be.

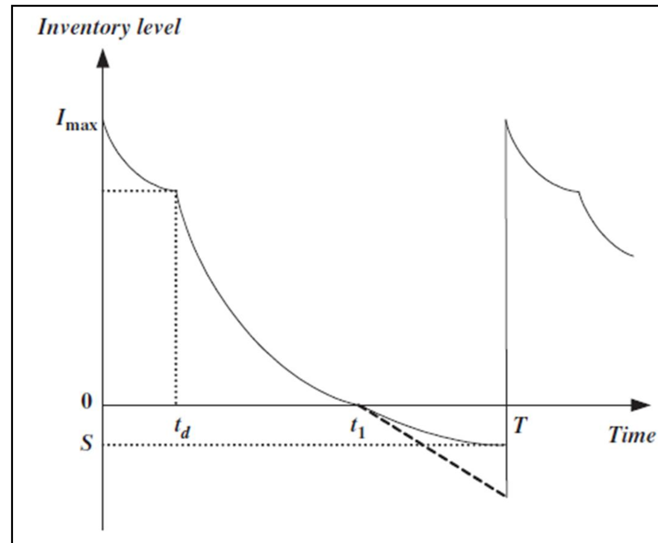


Figure 11: Graphical representation of inventory system, Wu, et al. (2006).

The outcome shows that the retailer can reduce total annual relevant inventory cost by ordering lower quantity when the supplier provides a permissible delay in payments, raising the length of time in which the product has no deterioration, or improving storage conditions for non-instantaneous deteriorating items. After that Chung (2009) completes the proof for the developed theorems in Oyang, et.al (2006).

Huang and Liao (2008) introduce a simple method to find the optimal ordering policy for deteriorating inventory under the assumption of permissible delay in payment for constant demand and no shortages were allowed and when deteriorating rate is exponential, and during the time the account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of this period the customer pays the supplier the total amount in the interest bearing account, and then starts paying off the amount owed to the supplier whenever the customer has money obtained from sales.

Shah and Mishra (2010) relaxed the EOQ model to include deteriorating inventory with constant deteriorating rate when demand is stock dependent with no shortages and zero lead time. In their model they assume that:

- The inventory system deals with single item.
- The demand for the item is stock dependent.
- Shortages are not allowed and lead time is zero.
- Replenishment rate is infinite. Replenishment is instantaneous.
- During the time account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of period the customer pays off all units sold, keep profits and starts paying for the interest charges on the items on stocks.
- Time horizon is infinite.
- The following notations are used in the formulation of the model:
- $R(Q(t)) = \alpha + \beta Q(t)$: the demand rate per annum (stock dependent), where α (> 0) is fixed demand and β (> 0) denotes stock dependent parameter. also $\alpha >> \beta$.

C : the unit purchase cost.

P: unit price.

γ C : salvage value of deteriorated unit, $0 \le \gamma < 1$.

h : the inventory holding cost per unit per year excluding interest charges.

A : ordering cost per order.

M : period of cash discount.

N : period of permissible delay in setting the account with N > M

I_c : the interest charged per $ in stock per year by the supplier or a bank.

I_e : the interest earned per $ in stock per year

r: cash discount ($0 < r < 1$).

θ : the constant deterioration rate. $0 < \theta < 1$.

Q : the procurement quantity (a decision variable)

T : the replenishment cycle time (a decision variable). And T^* is the optimal value of replenishment cycle time.

Q(t) : the on - hand inventory level at any instant of time t, $0 \le t \le T$.

D(T) : the number of units deteriorated during the cycle time T.

K(T) : the total inventory cost per time unit is the sum of : ordering cost OC, inventory holding cost (excluding interest charges) IHC, cost due to deterioration CD, cash discount earned if payment is made at M is DS, salvage value of deteriorated units SV, and cost of interest charges for unsold items after the permissible delay M or N, interest earned from sales revenue during the permissible period [0, M] or [0, N], and according to the above assumptions Shah and Mishra (2010) developed the next mathmatical model:

$$Q(t)= \alpha[e^{(\theta+\beta)T}-1]/(\theta+\beta)$$

$$\text{Ordering Cost } OC=A/T$$

$$\text{Cost of deteriorating units } CD= C\alpha[e^{(\theta+\beta)T}-(\theta+\beta)T-1]/(\theta+\beta)T$$

$$\text{Salvage value of deteriorated units } SV= \gamma C\alpha[e^{(\theta+\beta)T}-(\theta+\beta)T-1]/ (\theta+\beta)T$$

$$\text{Inventory holding cost } IHC =h\alpha[e^{(\theta+\beta)T}-(\theta+\beta)T-1]/ (\theta+\beta)^2T$$

Case 1: The payment is made at M and N to get a cash discount and M ≤ T. (see Figure 12).

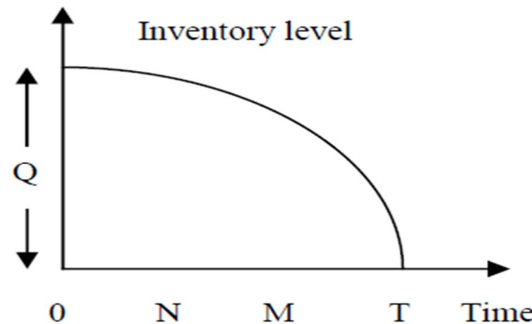


Figure 12: Inventory – time representation when M ≤T, Shah and Mishra (2010).

For this case

Discount saving per year; DS= rCQ/T

Interest payable per year for case 1is: $IC_1= CI_c\alpha[[e^{(\theta+\beta)(T-M)}-(\theta+\beta)(T-M)-1]/ (\theta+\beta)T$

Interest earned per year for case 1is: $IE_1= PI_e\alpha\theta M^2/2(\theta+\beta)T+pI_e\alpha\beta e^{(\theta+\beta)T}/(\theta+\beta)^3T$

Total cost per time unit for case 1is: $K_1 (T) = OC+CD+IHC-DS+IC_1-IE_1-SV$

The optimum value of $T=T_1$ is the solution for non linear equation $\partial K_1(T)/\partial T=0$,where:

$T=T_1=\sqrt{2A+\alpha M^2[cI_c-PI_e]/\alpha[h+C(1-\gamma-r)(\theta+\beta)+CI_c+PI_e\beta M]}$

Case 2: The retailer pays in full at M to get a cash discount but T < M. (See Figure 13).

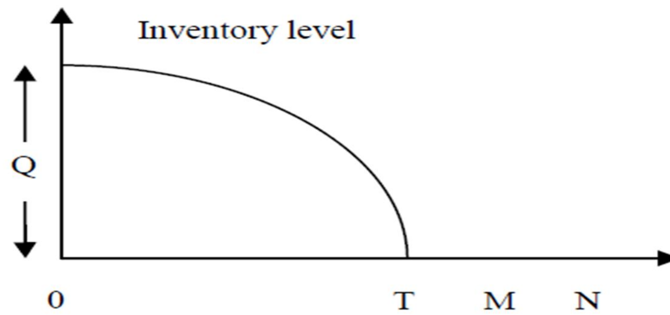


Figure 13: Inventory – time representation when T < M, Shah and Mishra (2010).

Interest earned per year for case 2 is:

$IE_2= PI_e[\alpha\theta T^2/2(\theta+\beta) +\alpha\beta[e^{(\theta+\beta)T}-(\theta+\beta)T-1]/ (\theta+\beta)^3]$

Total cost per time unit for case 2 is: $K_2 (T) = OC+CD+IHC-DS+IC2-IE2-SV$

The optimum value of $T=T_2$ is the solution for non linear equation $\partial K_2(T)/\partial T=0$,where:

$T=T_2=\sqrt{2A/\alpha[h+C(1-\gamma-r)(\theta+\beta)+PI_e]}$

Case 3: The payment is made at N to get the permissible delay and N ≤ T. (See Figure 14).

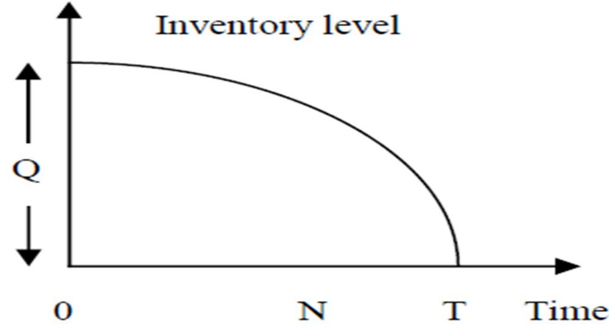


Figure 14: Inventory – time representation when $N \leq T$, Shah and Mishra (2010).

Interest payable per year for case 3 is: $IC_3 = CI_c\alpha[e^{(\theta+\beta)(T-N)} - (\theta+\beta)(T-N)-1]/(\theta+\beta)^2T$

Interest earned per year for case 3 is:

$IE_3 = PI_e\ \alpha[\theta N^2/2+\beta[-N(\theta+\beta)Te^{(\theta+\beta)(T-N)}+e^{(\theta+\beta)T}]/(\theta+\beta)^2]/(\theta+\beta)T$

Total cost per time unit for case 3is: $K_3(T) = OC+CD+IHC-DS+IC_3-IE_3-SV$

The optimum value of $T=T_3$ is the solution for non linear equation $\partial K_3(T)/\partial T=0$,where:

$$T=T_3=\sqrt{2A+\alpha N^2[cI_c - PI_e]/\alpha[h+C(1-\gamma-r)(\theta+\beta)+CI_c+PI_e\beta N]}$$

<u>Case 4:</u> The retailer pays in full at N but T < N. (See Figure 15).

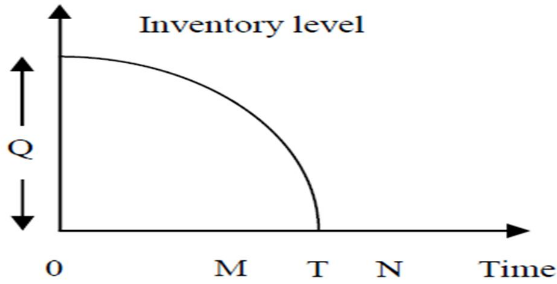


Figure 15: Inventory – time representation when T ▪ N, Shah and Mishra (2010).

Interest earned per year for case 4 is:

$IE_4 = pI_e[\alpha\theta T^2/2(\theta+\beta) + \alpha\beta\ (e^{(\theta+\beta)T}-(\theta+\beta)T-1)+\alpha T(N-T)/(\theta+\beta)^3]$

Total cost per time unit for case 4 is; $K_4\ (T) = OC+CD+IHC-DS+IC_4-IE_4-SV$

The optimum value of $T=T_4$ is the solution for non linear equation $\partial K_4(T)/\partial T=0$,where:

$$T=T_4=\sqrt{2A/\alpha[h+C(1-\gamma-r)(\theta+\beta)+PI_e]}$$

Where T_1,T_2,T_3 and T_4 are the optimal values for cases 1,2,3 and 4 respectively.

And to solve the above model they developed the following algorithm:

If $2A > M^2 \alpha [h + C (1 - \gamma - r) (\beta + \theta) + P I_e \beta M + P I_e]$, then $T^* = T_1$.

If $2A = M^2 \alpha [h + C (1 - \gamma - r) (\beta + \theta) + P I_e \beta M + P I_e]$, then $T^* = M$.

If $2A < M^2 \alpha [h + C (1 - \gamma - r) (\theta + \beta) + P I_e]$ then $T^* = T_2$.

If $2A > N^2 \alpha [h + C (1 - \gamma) (\beta + \theta) + P I_e \beta N + P I_e]$ then $T^* = T_3$.

If $2A = N^2 \alpha [h + C (1 - \gamma) (\beta + \theta) + P I_e \beta N + P I_e]$ then $T^* = N$.

If $2A < N^2 \alpha [h + C (1 - \gamma) (\theta + \beta) + P I_e]$ then $T^* = T_4$.

If $M^2 \alpha[h+C(1-\gamma-r)(\beta+\theta)+PI_e \beta M+P I_e] < 2A < N^2\alpha[h+C(1 -\gamma)(\theta+\beta)+P I_e]$ then

If $K_1(T_1) < K_4(T_4)$ then $T^* = T_1$ otherwise $T^* = T_4$.

Based on the above algorithm they found that as deterioration decreases optimal time decreases whereas total cost increases. Cycle time decreases as stock dependent parameter increases moreover total cost increases at the same time. With increase in cash discount rate cycle time increases but total cost decreases and cycle time increases as salvage parameter increases but total cost of inventory decreases.

Geetha and Uthayakumar (2010) developed an EOQ model for non-instantaneous deteriorating items with permissible delay in payment and allowed shortages with partial back logging. Here backlogging rate is considered to depend on waiting time for the next replenishment. The following assumptions and notations were used throughout their model:

(1) D is the annual demand rate for the item is constant.

(2) Replenishment rate is infinite and lead time is zero.

(3) Shortages are allowed and partially backlogged. It is to be noted that, the longer the waiting time is, the smaller the backlogging rate will be. Let B(T) denote this fraction where t is the waiting time up to the next replenishment. $B(t)= 1/(1+\delta t)$, where δ is the backlogging parameter $0\leq\delta\leq 1$.

(4) The product life (time to deterioration) t has a probability density function f (t)= 1-e-θ(t-td) for $t>t_d$ where t_d is the length of time in which the product has no deterioration (fresh product time) and θ is a parameter. The cumulative distribution function of t is given by F(t)= $\Box^{t}_{td}$f(x) dx=1-e-θ(t-td) for t>td so that deterioration rate is r(t) = f(t)/1-F(t)=θ , for $t>t_d$

(5) t_d can be estimated by utilizing the random sample data of the product during past time and statistical maximum likelihood method. For simplicity, we assume that td is a given constant and $t_d \leq t_1$.

(6) During the trade credit period, M, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the period, the retailer pays off all units bought, and starts to pay the capital opportunity cost for the items in stock.

(7) The system operates for an infinite planning horizon.

The following were the notations used:

A: the ordering cost per order.

h: Holding cost per unit per year excluding interest charges.

p: The purchasing cost per unit.

p_1: The selling price per unit.

s :The shortage cost for backlogged items per unit per year.

π:The unit cost of lost sales per unit.

I_p: The capital opportunity cost in stock per dollar per year.

I_e: The interest earned per dollar per year.

T: Length of order cycle.

t_1: Length of time in which the inventory has no shortage.

Q: The order quantity per cycle.

TC(t_1, T): The total annual inventory cost.

M: trade credit period. Geetha and Uthayakumar (2010) developed the following model based on the above assumptions and notations (See Figure 16):

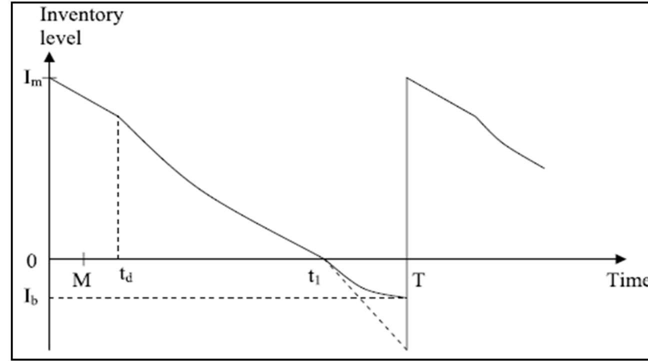


Figure 16(a): Case 1, 0 ▪ $M \le t_d$.

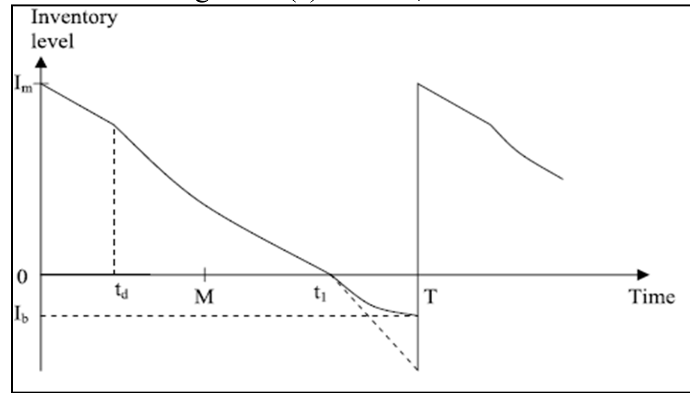


Figure 16(b): Case 2.

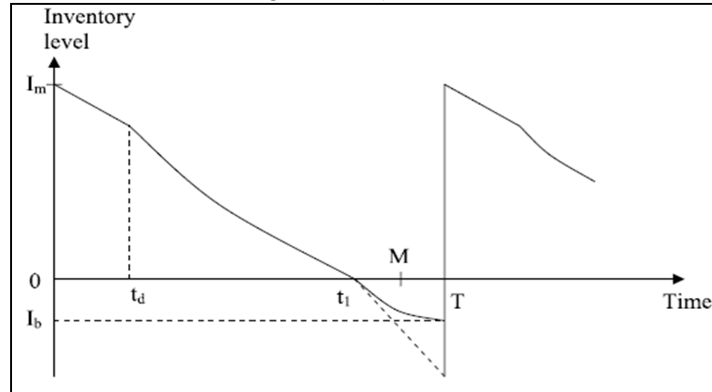


Figure 16(c): Case 2, $M > t_1$.

Figure 16(a,b,c): Graphical representation for Geetha and Uthayakumar (2010) inventory system.

$$TC(t_1,T)= \begin{cases} TC_1(t_1,t) & \text{if } 0 ▪ M \le t_d \\ TC_2(t_1,t) & \text{if } t_d ▪ M \le t_1 \\ TC_3(t_1,t) & \text{if } M > t_1 \end{cases}$$

Where,

$$TC_1(t_1,t)=\frac{D}{T}\Big\{A/D+\frac{ht_d}{\theta}[e^{\theta(t_1-t_d)}-1]+\frac{ht_d^{\;2}}{\theta^2}+\frac{(h+p\theta)}{\theta^2}[e^{\theta(t_1-t_d)}-\theta(t_1-t_d)-1]$$

$$+\frac{(s+\delta\pi)}{\delta}[(T-t_1)-(\frac{\ln[1+\delta(T-t_1)]}{\delta})]+\frac{pI_p}{\theta^2}[e^{\theta(t_1-t_d)}-1]+\frac{(td-M)^2}{2}$$

$$+\frac{1}{\theta^2}[e^{\theta(t_1-t_d)}-1-\theta(t_1-t_d)]\ \Big\}-\frac{p_1I_eM^2}{2}\ \}$$

$$TC_2(t_1,t)=\frac{D}{T}\Big\{A/D+\frac{ht_d}{\theta}[e^{\theta(t_1-t_d)}-1]+\frac{ht_d^{\;2}}{\theta^2}+\frac{(h+p\theta)}{\theta^2}[e^{\theta(t_1-t_d)}-\theta(t_1-t_d)-1]$$

$$+\frac{(s+\delta\pi)}{\delta}[(T-t_1)-(\frac{\ln[1+\delta(T-t_1)]}{\delta})]+\frac{pI_p}{\theta^2}[e^{\theta(t_1-M)}-\theta(t_1-M-1]-\frac{p_1I_eM^2}{2}\ \Big\}$$

$$TC_3(t_1,t)=\frac{D}{T}\Big\{A/D+\frac{ht_d}{\theta}[e^{\theta(t_1-t_d)}-1]+\frac{ht_d^{\;2}}{\theta^2}+\frac{h+p\theta}{\theta^2}[e^{\theta(t_1-t_d)}-\theta(t_1-t_d)-1]$$

$$+\frac{(s+\delta\pi)}{\delta}[(T-t_1)-\frac{\ln[1+\delta(T-t_1)]}{\delta}\,p_1I_et_1(M-\frac{t_1}{2})\ \Big\}$$

Based on the developed model they found that the optimal cycle time for each case is:

Case1:0≤M≤td

$$T=t_1+\frac{(L+V)e^{\theta(t1-td)}-(U+W)}{\delta[(U+W)+N-(L+V)e^{\theta(t_1-t_d)}]}$$

Case 2: td≤M≤t1

$$T=t_1+\frac{Le^{\theta(t1-td)}+Ue^{\theta(t1-M)}-(W+U)}{\delta[(U+W)+N-Le^{\theta(t1-td)}+Ue^{\theta(t1-M)}]}$$

And for case 3: M>t1

$$T=t_1+\frac{Le^{\theta(t1-td)}-W+p_1I_e(t_1-M)}{\delta[W-Le^{\theta(t1-td)}-p_1I_e(t_1-M)+N]}$$

Where $N=\dfrac{s+\delta\pi}{\delta}$, $L=ht_d+W$, $W=\dfrac{h+p\theta}{\theta}$, $U=\dfrac{pI_p}{\theta}$ and $V=pI_p(t_d-M)+U$

Based on the above model they concluded that when the fresh product time increases and other parameters remain unchanged, the optimal total annual cost decreases. That is, the longer the fresh product time is, the lower total cost would be. It implies that the model with non-instantaneous deteriorating items always has smaller total annual

inventory cost than with instantaneous deteriorating items and if the retailer can extend effectively the length of time the product has no deterioration for a few days or months, the total annual cost will be reduced obviously. Increasing the fresh product time (t_d) decreases the order quantity (Q). Also from the inventory point of view, the longer fresh product time is, the lower-order quantity would be. It can be found that optimal value for t_1 and T increases with an increase in t_d, which implies that the longer the fresh product time is, the longer the replenishment cycle and the length of inventory interval with positive inventory. It can be found that each of optimal value of T , Q and TC decreases with an increase in the credit period (M) (other parameters are fixed) which implies that the longer the credit period is the shorter the replenishment cycle, the lower the order quantity and the total annual cost will be. From economical point of view, if the supplier provides a permissible delay in payments, the retailer will order lower quantity in order to take the benefits of the permissible delay more frequently. Also they found that Increasing the backlogging parameter (δ) (or equivalently decreasing the backlogging rate) decreases the optimal order quantity Q and increases the optimal total annual cost TC that indicates that when shortages are completely backlogged, total cost per unit time becomes lower. Also the replenishment cycle time decreases with an increase in the backlogging parameter (δ). It can be seen that when the parameter θ increases, the optimal value of T, t_1 and Q decrease while TC increases. Hence, if the retailer can effectively reduce the deteriorating rate of item by improving equipment of storehouse, the total annual inventory cost will be lowered. Also they found that the optimal values of TC, Q, T and t_1 decreases with the increase in the value of the parameter I_e. That is, total annual cost, order quantity, the length of replenishment cycle and the length of inventory interval with positive inventory decreases with increase in I_e, this implies that when the interest earned per dollar is high the total cost is low, and

the Increase in I_p results in a decrease in, T, Q and t_1 and an increase in TC. The total cost increases when the capital opportunity cost in stock per dollar is high. From managerial point of view it implies that when the capital opportunity cost in stock per dollar is high the retailer should order less amount of inventory.

Basu and Sinha(2007) presents general inventory model with assumptions that inventory deteriorate rate is time dependent and has a weibull distribution, backlogging is partial and depends on waiting time, also demand rate is time dependent with permissible delay in payment also they studied the impact of inflation on their model. They assume that the inventory system involves only one item. The rate of replenishment is instantaneous. A fraction z(t) of the on hand inventory deteriorates per unit time where shortages are allowed and partially backlogged, and the demand rate R(t) at any time t is given by R(t) = a + bt where a and b are non-negative constants. They derived theorems to solve the model, which can help the decision maker to determine the optimal cycle time and to minimize the total average inventory cost. And they used the following notations in order to develop their model they used the following notations: q(t) = Inventory level at time t, Inflationary inventory model with time dependent demand:

S = q(0) = Stock level at the beginning of each cycle after fulfilling backorders

H = Length of the planning horizon

K = Constant rate of inflation ($/$/ unit time)

C(t) = Unit purchase cost for an item bought at time t, i.e., $C(t) = C_o.e^{KT}$, where Co is the unit purchase cost at time zero

h = Holding cost ($/unit/year) excluding interest charges

C_o = Unit purchase cost

C_2 = Shortage cost ($/unit/time)

C_3 = the ordering cost/cycle

i_e = Interest earned ($/time)

i_p = Interest charged ($/time)

M = Permissible delay in settling the accounts

T_1 = Time at which shortages start ($0 \le T_1 \le T$)

T = Length of a cycle

TCU(T_1, T) = The average total inventory cost per unit time

$TCU_1(T_1, T)$ = The average total inventory cost per unit time for $T_1>M$ **(Case I)**

$TCU_2(T_1, T)$ = The average total inventory cost per unit time for $T_1 \le M$ **(Case II)**

And they developed the following model:

Case I: $T_1>M$ (payment before depletion)

For this case the average total inventory cost per unit time is:

$$
\begin{aligned}
TCU_1 = {} & C_0\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\left[\frac{\alpha T_1^{\,2}}{2}(h-i_c-i_p)+\frac{bT_1^{\,3}}{6}(2h-i_e-i_p)+C_3\right. \\
& +\alpha T_1^{\,\beta+3}\left\{\frac{2\beta^2+4\beta(b+1)+8b+b\beta i_p}{2(\beta+2)(\beta+3)}-\frac{b}{2\beta+1}\right\} \\
& +\left\{\frac{\alpha T_1^{\,\beta+2}}{\beta+2}\left\{\frac{\beta(ah+ai_p+b)+b}{\beta+1}\right\}+C_2\left(\frac{a\delta+b+b\delta T}{\delta^2}\right)\left\{\frac{1}{\delta}\ln|T-T_1|-(T-T_1)\right\}\right\} \\
& +\frac{bC_2}{2\delta}(T-T_1)^2+\frac{a\alpha T_1^{\,\beta+1}}{\beta+1}+i_p\left(aT+\frac{bT_1^{\,2}}{2}+\frac{a\alpha T_1^{\,\beta+1}}{\beta+1}+\frac{b\alpha T_1^{\,\beta+2}}{\beta+2}\right)\times\left\{(T_1-M)-\frac{\alpha}{\beta+1}(T_1^{\,\beta+1}-M^{\beta+1})\right\} \\
& \left.+\frac{ai_pM^2}{2}+\frac{bipM^3}{6}-\frac{ai_p\alpha\beta M^{\beta+2}}{(\beta+1)(\beta+2)}\right]/T
\end{aligned}
$$

Case II: $T_1 \cdot M$ (payment after depletion)

$$
\begin{aligned}
TCU_2 = {} & C_0\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\left[\frac{bT_1^{\,3}}{2}(2h-i_c-3)+C_3+\frac{T_1^{\,2}}{2}\{a(h-i_c-2)+bM\}+aMT_1\right. \\
& +\frac{bC_2}{2\delta}(T-T_1)^2+\alpha T_1^{\,\beta+3}\left\{\frac{\beta^2+2\beta(b+1)+4b}{(\beta+2)(\beta+3)}-\frac{b}{2(\beta 1)}\right\}+\frac{a\alpha T_1^{\,\beta+1}}{\beta+1}+\frac{\alpha T_1^{\,\beta+2}\{\beta(ah+b)+b\}}{(\beta+1)(\beta+2)} \\
& \left.+C_2\,\frac{(a\delta+b+b\delta T}{\delta^2})\left\{\frac{1}{\delta}\{\ln|T-T_1|-(T-T_1)\}\right\}\right]/T
\end{aligned}
$$

To solve the above model Basu and Sinha (2007) used Genetic Algorithm as a procedure to solve the numerical example. From the sensitivity analysis, it was inferred that as the rate of deterioration increases, the total average inventory cost increases, which is obvious. Moreover, it follows that increase in permissible delay decreases total cost which means that there is an inverse relation between total cost and the permissible payment period. The intuitive reason behind this is that the extension in permissible payment period offers opportunity to the purchaser to earn more by investing the resource otherwise from the sale-proceed of the inventory, which results in the lower cost.

2.3 Inventory systems for deteriorating inventory with backorders

In most inventories when items are deteriorate back logging may happened and it may fully or partially backlogged therefore Dye, et al. (2006) introduce an inventory model where backlogging rate of unsatisfied demand linearly depends on the total number of customer in waiting line during shortage period. In their model, lead time is assumed to be zero and deterioration rate constant. They added the cost of lost sales and the total purchase cost into the model, and therefore establish an appropriate model for a retailer to determine its replenishment number and schedule when the backlogging rate is linearly dependent on the total number of customers in the waiting line. The proposed model allows not only the partial backlogging rate to be linearly dependent on the total amount of backorders but also a constant deterioration rate, (See Figure 17), their model is:

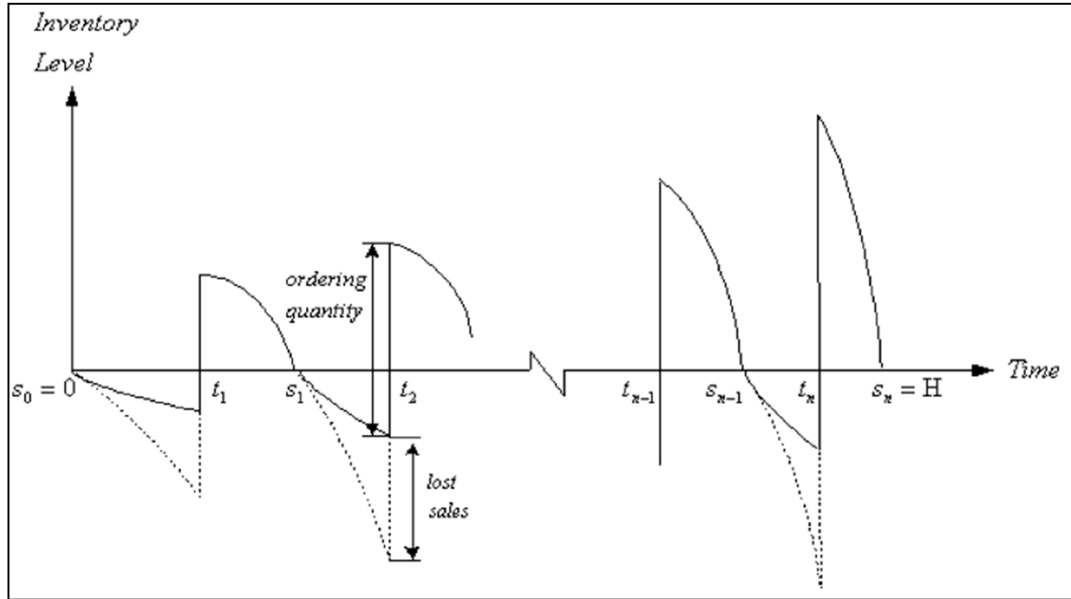


Figure 17: Graphical representation of inventory system, Dye, et al. (2006).

And the total profit for this inventory system is:

$$TP(n,\{s_i\},\{t_i\}) = (p-c_v)\sum_{i=1}^{n}\left[\int_{s_{i-1}}^{t_i} e^{-\delta(t_i-t)} f(t)dt + \int_{t_i}^{s_i} f(t)dt\right] - nc_f - \frac{c_h + c_v\theta}{\theta}\sum_{i=1}^{n}\int_{t_i}^{s_i}[e^{\theta(t-t_i)}-1]f(t)dt$$
$$-\frac{c_s+\delta(p-c_v+c_g)}{\delta}\sum_{i=1}^{n}\int_{s_{i-1}}^{t_i}[1-e^{-\delta(t_i-t)}]f(t)dt$$

Where, H is the planning horizon of the inventory problem here is finite and is taken as H time units, θ the deterioration rate, c_f is the fixed purchasing cost per order, p is the selling price per unit, f(t) is the demand rate at time t, where f(t) is positive and log-concave in the planning horizon (0,H],c_v is the variable purchasing cost per unit, c_h the inventory holding cost per unit per unit time, c_s is the backlogging cost per unit per unit time due to shortages, cg is the cost of lost goodwill, c_l the unit cost of lost sales. Note that if the objective is to minimize the costs, then $c_l = p + c_g > c_v$. If the objective is to maximize the profits, then $c_l = c_g$. n is the number of replenishments over [0,H] (a decision variable), t_i is the ith replenishment time (a decision variable), i = 1,2, . . . , n.

s_i is the time at which the inventory level reaches zero in the ith replenishment cycle (a decision variable),i = 1,2, . . . , n. T_i is the length of the ith cycle, Q_i is the order quantity at t_i in the ith replenishment cycle and TP is total profit for the inventory system.

To solve their model Dye, et al.(2006) proposed a simple and effective algorithm with 2n + 1 decision variables, where the estimated value for number of replenishments (n) is: $n = \sqrt{\dfrac{(c_v + c_v\theta)[e^{-\delta}c_s + [(1-e^{-\delta})(p - c_v + c_l)]Q(H)H}{2c_f[c_h + c_v\theta + e^{-\delta}c_s + (1-e^{-\delta})(p - c_v + c_l)}}$

Furthermore, the demand function used in this model is quite general and gives much flexibility to cover many demand scenarios. You and Hsieh (2007) investigates a production planning problem where inventory deteriorate at constant rate, demand is price dependent, backorders are allowed and customer may cancel backorder before receiving them (See Figure 18). The following notations were used: L is the production lead time, T is the cycle time, $T = T_1 + L + T_4$; $I(t)$ is the inventory level at time t, I_{max} is the maximum inventory level, I_b is the maximum stock-out demand, s is the selling price, $d(s)$ is the demand rate when a sale price is set at s (the demand rate $d(s)$ is assumed to be a linear function of $d(s) = a - b_s$ where a and b are (constant values), p is the production rate per unit time, θ is the inventory deteriorating rate per unit time, β is the fraction of the number of cancellation, λ is the fraction of revenue from penalty for cancellation to the selling price, *s,* (the revenue from penalty for cancellation is λs), c_s is the setup cost per production run, c_b is the unit backordering cost per unit time, c_d is the deteriorated cost per unit *and* c_h is the unit inventory carrying cost per unit time.

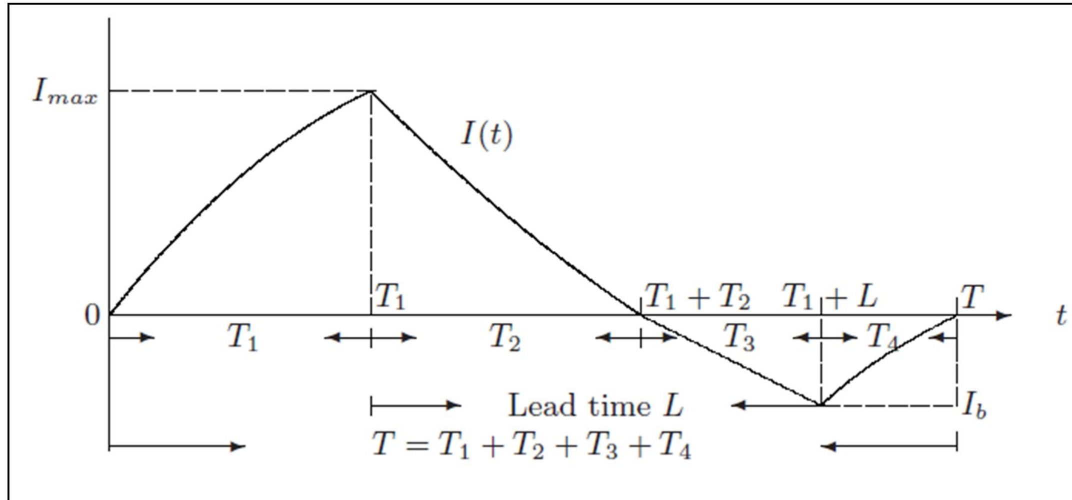


Figure 18: Behavior of deteriorating inventory level with time, You and Hsieh (2007) .

In their model the profit per unit time is:

$$R = TR/T = (R_s + R_c - K_s - K_h - K_d - K_b)/T$$

Where:

Production cost= K_s=c_s

Inventory holding cost = $K_h = c_h(\int_0^{T_1} I_1(t)dt + \int_0^{T_2} I_2(t)dt)$

Back-ordering cost= $K_b = c_b(-(\int_0^{T_3} I_3(t)dt + \int_0^{T_4} I_4(t)dt))$

$$K_b = c_b(\frac{d(s)(T_3 + T_4) - pT_4}{\beta})$$

Deteriorating cos= $K_d = c_d((T_1(p - d(s)) - I_{max}) + (I_{max} - d(s)(T_2 - T_1))$

$$K_d = c_d((p - d(s))T_1 - d(s)T_2)$$

Revenew from cancellation= $R_c = \lambda s(d(s)(T_1 + T_2) - pT_4)$

Sales revenue= $R_s = s(d(s)(T_1 + T_2) - pT_4)$

Based on the assumption that $I_1(T_1)$=$I_2(0)$ thus $T_1 = \frac{-1}{\theta}\ln(\frac{p - d(s)e^{\theta T_2}}{p - d(s)}$,

And based on the assumption that $I_3(T_3)=I_4(0)$, $T_4 = \frac{1}{\beta}\ln(\frac{p-d(s)e^{\beta T_3}}{p-d(s)})$, $T_3=L-T_2$.

2.4 Stochastic inventory systems with delay in payment

All previous mentioned researches dealt with deterministic environment, and assume that lead-time demand is zero or constant but in general; the control parameters (Q and R) depend on both the demand process and the replenishment lead time. Although many studies have treated lead time as constant, focusing solely on demand (d) in (Q, R) model with stochastic lead time could be a building block in Supply Chain Management. Variability in lead time between successive stages is often what disturbs supply chain coordination. For that, Bookbinder and Cakanyildirim (1999) studied a two-stage system with a constant demand rate, they concentrated on lead time as a random variable, and developed two probabilistic models. In the first, lead time T is exogenous, the second model lead time T is endogenous, thus has three decision variables for each model, they show that the expected cost per unit time is jointly convex in the decision variables and obtain the global minimize. They then established joint convexity of the objective function in decision variables Q and R, and conducted sensitivity analyses with respect to the model's cost parameters. An exponentially distributed lead time could in principle cause a realized value to be unbounded. A lower value of the expediting factor s would achieve the same effect as shortening the tail of the lead time distribution. The latter point indicates how order crossing would be countered in practice, namely by investing in expedited lead times. This in turn would increase the validity of their assumption, i.e. that upon arrival of a lot (Q units), installation stock is always above the reorder level (See Figure 19). Such products are not necessarily stocked at a given warehouse, but rather flow directly to the

next lower level (e.g. to retailers). However, not every item, nor even most items, will be handled in a strictly flow through approach. Those items will be stocked somewhere, and this models are applicable to orders on that location in the supply chain. The probability distribution of lead time from there to the next lower echelon where the product is stocked is the effective lead time relevant to these models.

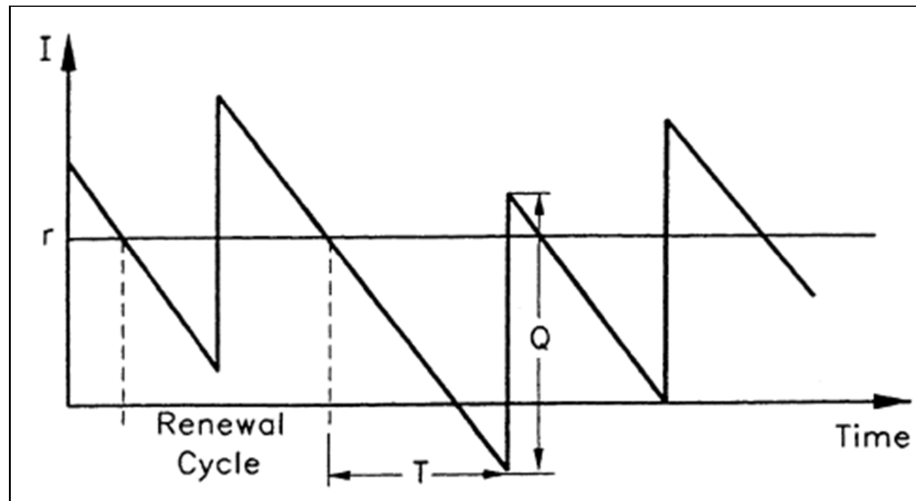


Figure 19: Installation stock in the case of complete backordering. T is a realized lead time of the random variable, Bookbinder and Cakanyildirim (1999).

In fact, the key distribution is well known to be that of lead time demand (LTD), the convolution of the lead time distribution with the demand distribution. With no expediting, the reorder point can be related to LTD in each case: demand random, constant lead time; random lead time, constant demand; or both random variables. With expediting, the relevant p.d.f. should still be that of LTD. By assuming that demand was constant, they made more transparent the implications of expedited lead times. Wu (2001) dealt with probabilistic inventory environment to find the optimal continuous review (Q, R) inventory policy under delay in payment. In his study lead time demand considered to be normally distributed, backorders are allowed and completely backlogged. In his model he aims to find the optimum order quantity and replenishment point that minimizes the total inventory cost, where he derived theorems and applies

two solution procedures the first was a simple algorithm and the second was a distribution free procedure:

The total expected annual cost function is:

$$EAC(Q,R) = \frac{AD}{Q} + pD + \frac{hQ}{2} + (h + pI_c)(R - \mu) + \frac{\pi D}{Q} n(R)$$
$$- \frac{D^2 pt_c^{\ 2} I_d}{2Q} - n(R) pt_c I_d \frac{D}{Q} + \frac{(Q - Dt_c)^2 pI_c}{2Q}$$

where A is the fixed order cost, D is the annual demand, p is the unit cost of item, h is the unit holding cost, μ is the mean lead-time demand, π is the unit shortage cost. x is the demand during the lead-time, F(x) the cumulative distribution function of the lead-time demand, and $n(R) = \int_R^{\infty 0} (x - R) df(x)$, Moreover, Q/2 is often called the cycle stock and (R -μ) is called the safety stock, he considered the effect of credit period on the above model. Let t_c be the credit period and h is redefined as the unit stock-holding cost per item per year excluding the interest charges for financing the stock. Let I_d be the rate at which the buyer from the sales amount derives interest and I_c be the interest rate applicable to the stock value after the credit period $I_c \geq I_d$). The holding cost for this stock is (h+pI_c) (R -μ), and the holding cost for the cycle stock is $hQ^2/2D$, by finding the first derivative of the total expected annual cost function with respect to, Q, the order quantity, and with respect to, R, the replenishment point and equating the derivatives to zero the value of Q and R will be:

$$Q = \sqrt{\frac{2D[A + (\pi - pt_c I_d n(R) + 0.5 D t_c^{\ 2} p(I_c - I_d)]}{h + pI_c}}$$

$$1 - F(R) = \frac{(h + pI_c)Q}{(\pi - pt_c I_d)D}$$

To find the optimal ordering quantity and replenishment point Wu (2001) developed the following algorithm:

Step 0: use $EOQ = \sqrt{\frac{2AD}{h}}$ as the initial estimate for Q. Call this value Q_0. Let i = 0.

Step 1: Use equation $1 - F(R) = \frac{(h + pI_c)Q}{(\pi - pt_c I_d)D}$ with Q = Qi to find the reorder point R.

Call this value R_i.

Step 2: Use equation $Q = \sqrt{\frac{2D[A + (\pi - pt_c I_d n(R) + 0.5Dt_c^2 p(I_c - I_d)]}{h + pI_c}}$ with $R = R_i$ to

find Q_{i+1}. If $Q_i = Q_{i-1}$ and $R_i = R_{i-1}$ then stop. Otherwise, set i = i + 1, go to Step 1.

His model results that the optimal order quantity and optimal reorder point are less sensitive to changes in the demand rate and unit cost. But, the minimum expected annual cost is highly sensitive to changes in the value of rate and unit cost.

Chang (2002) extended Wu's 2001 model to include the ordering cost as one of the decision variables. Chang (2002) explore the (Q, R) inventory policy as well as the investment strategy for ordering cost reduction under conditions of permissible delay in payments, where the relationship between ordering cost and its investment is formulated by the widely utilized logarithmic function. He first considers a case where the lead-time demand is normally distributed, and develops an algorithm to find the optimal solution. Then, the Laplace distribution is employed to model the lead-time demand, for this case, he derived the closed-form solution and perform the sensitivity analysis. He extend Wu's (2001) model to include ordering cost as one of the decision variables, where the logarithmic investment-cost function is utilized to describe the relationship between ordering cost and its investment. The problem which was explored is: when a delay in payments is permitted, how much is worth investing in reducing ordering cost; and how the credit period influences the investment strategy for ordering cost reduction. He found that the longer the credit period offered by the supplier, the less the investment will be made to reduce the ordering cost. The logarithmic

investment-cost function is adopted by Chang (2002) to describe the relationship between ordering cost and its investment.

In our study we extended Wu (2001) model to include deterioration inventory and partially backlogged backorders.

2.5 Genetic Algorithm (GA)

Genetic algorithms are a part of evolutionary computing, which is a rapidly growing area of artificial intelligence. They are a class of adaptive search techniques based on the principle of population genetics. GA work according to the principles of natural genetics on a population of string structures representing the problem variables. All these features make genetic algorithm search robust, allowing them to be applied to a wide variety of problems (Moghadam, et al., 2008)

Genetic algorithms has been used for difficult problems (such as NP-hard problems), for machine learning and also for evolving simple programs. They have been also used for some art, for evolving pictures and music and in inventory control and supply chain management.

Advantage of GAs is in their parallelism. GA is travelling in a search space with more individuals (and with genotype rather than phenotype) so they are less likely to get stuck in a local extreme like some other methods.

They are also easy to implement. Once you have some GA, you just have to write new chromosome (just one object) to solve another problem. With the same encoding you just change the fitness function and it is all. On the other hand, choosing encoding and fitness function can be difficult.

Mondal and Maiti (2002) proposed soft computing approach to solve non-linear programming problems under fuzzy objective goal and resources with/without fuzzy parameters in the objective function. They used genetic algorithms (GAs) with mutation and whole arithmetic crossover, where mutation is carried out along the weighted gradient direction using the random step lengths based on Erlang and Chi-square distributions. These methodologies have been applied to solve multi-item fuzzy EOQ models under fuzzy objective goal of cost minimization and imprecise constraints on warehouse space and number of production runs with crisp/imprecise inventory costs. The fuzzy inventory models have been formulated as fuzzy non-linear decision making problems and solved by both GAs and fuzzy non-linear programming (FNLP) method based on Zimmermann's approach, they found that for the fuzzy EOQ model with crisp inventory costs, FNLP gives the better result than the Gas indicating lowest expenditure and utilizing maximum possible storage space. Among two GA methods, GA based on Chi-square distribution is better. On the other hand, for fuzzy EOQ model with imprecise inventory costs, the picture is different. In this case, GA based on Erlang distribution is the best where here, both GA's give better results than FNLP. Radhakrishnan, et al. (2009) proposed an innovative and efficient methodology that uses Genetic Algorithms to precisely determine the most probable excess stock level and shortage level required for inventory optimization in the supply chain such that the total supply chain cost is minimal. Their problem was on a factory which is the parent of the chain and it is having two distribution centers Distribution center 1 and Distribution center 2. Each Distribution center further comprises of several agents but as stated in our exemplary case, each Distribution center is having two agents. So, in aggregate there are four agents, Agent1 and Agent2 for Distribution center 1 and Agent 3 and Agent 4 for

Distribution center 2. The factory manufactures different products that would be supplied to the distribution centers. From the distribution center, the stocks will be moved to the corresponding agents. Radhakrishnan, et al. (2009) determined the exact product that needs to be concentrated on and the amount of stock levels of the product to be maintained by the different members of the supply chain, also whether the stock level of the particular product needs to be in abundance in order to avoid shortage of the product or needs to be held minimal in order to minimize the holding cost. The proposed approach of genetic algorithm performance was evaluated using MATLAB 7.4. By following the proposed genetic algorithm based approach for inventory management, they had successfully obtained the optimized stock levels at all members of the supply chain. The complexity of increasing the number of products through the supply chain has also been resolved by their proposed approach. They determined the products due to which the members of the supply chain incurred extra holding or shortage cost. More specifically, the inventory is optimized in the whole supply chain regardless of the number of products and the number of members in the supply chain. The proposed approach of inventory management has achieved the objectives which are the minimization of total supply chain cost and the determination of the products due to which the supplier endured either additional holding cost or shortage cost.

Maiti, et al. (2009) studied an inventory model where demand of the item depends on selling price, lead-time is stochastic in nature, and retailer has to pay some advance payment at the time of ordering and is eligible for a price discount against extra advance payment. Shortages are completely backlogged and are met as soon as new order arrives. Here objective is to maximize the expected average profit. A GA, based on Roulette selection, whole arithmetic crossover and non-uniform

mutation is developed for the model and objective function is optimized using it. Results via GA are compared with a non-linear optimization technique in some particular cases. In a particular situation when demand is constant, closed form solution is obtained. they illustrated Results for the models with and without advance payment and they observed that profit is more when advance payment is allowed and price discount due to that is permitted. Moreover, an algorithm in C language for GA is developed and is used to verify the results obtained by a non-linear optimization technique. GA algorithm has been implemented in C-language and executed with different seeds of random number generators they observed that all these executions leads to the same optimum solution. From that it may be concluded that the GA solution is global optimum. It is observed that average profit increases with advance payment but it was seen that rate of increase of profit slowly decreases with the increase of percentage of advance payment. This phenomenon agrees with reality. They also observed that a wholesaler needs a large amount of investment capitals to meet all the retailers demand. In fact it is difficult for him to arrange such a large amount from bank-loan (due to lack of mortgage property). In this situation, he offers some opportunity to his retailers by giving some discount from his profit, if retailers deposit some extra amount of advance payment; retailer takes this opportunity to make some extra profit if he has some mortgage property to take loan from bank.

Gupta, et al. (2009) The purpose of this research is to solve the mixed integer constrained optimization problem with interval coefficient by a real-coded genetic algorithm (RCGA) with ranking selection, whole arithmetical crossover and non-uniform mutation for non-integer decision variables. In the ranking selection, as well as in finding the best solution in each generation of RCGA, recently developed modified definitions of order relations between interval numbers with respect to

decision-making are used. Also, for integer decision variables, new types of crossover and mutation are introduced. This methodology is applied to solve a finite time horizon inventory model with constant lead-time, uniform demand rate and a discount by paying an amount of money in advance. Moreover, different inventory costs are considered to be interval valued. According to the consumption of items during lead-time and reorder level, two cases may arise. For each case, the mathematical model becomes a constrained nonlinear mixed integer problem with interval objective. Our objective is to determine the optimal number of cycles in the finite time horizon, lot-size in each cycle and optimal profit.

Maiti (2011) used GA to make managerial decision for an inventory model of a newly launched product. It assumed that lifetime of the product is finite and imprecise (fuzzy) in nature. Here wholesaler/producer offers a delay period of payment to its retailers to capture the market. Due to this facility retailer also offers a fixed credit-period to its customers for some cycles to boost the demand, during these cycles; demand of the item increases with time with a decreasing rate, depending upon the duration of customers' credit-period. Models are formulated for both the crisp and fuzzy inventory parameters to maximize the present value of total possible profit from the whole planning horizon under inflation and time value of money. He transferred Fuzzy models to deterministic ones following possibility/necessity measure on fuzzy goal and necessity measure on imprecise constraints. In his model the inventory system involves only one item and the cost of raw materials as well as processing cost of an item changes rapidly and due to competitive market manufacturers very frequently change their product specifications with new features, packets and name. As a result, the time horizon was assumed as finite and imprecise in nature. Retailer offers a credit-period For the first time, Maiti (2011) developed an

inventory model incorporating customers' credit-period dependent dynamic demand, inflation and time value of money where lifetime of the product is imprecise in nature, where customers' credit-period is offered by the retailer for few cycles to increase demand. After withdrawal of credit period increased demand retains during the rest of the cycles. To make more realistic inventory model, here, customers' credit period was taken as a decision variables. The objective of his model is to introduce credit-linked dynamic demand of a product, especially when lifetime of the product is a fuzzy number and to Study the effect of customers' credit-period on the demand under inflation and time value of money. He found that this GA is efficient in solving the proposed inventory model. And for more detail about the GA see appendix 1.

Chapter 3: Methodology

First step a literature survey was conducted about the deteriorate inventory, delay in payment and order cancellation and about using genetic algorithm to find the minimum cost of inventory models. Then, the inventory control model was developed for the above mentioned problems, and the equations of the optimum order quantity and the replenishment point under which the total cost is minimum was developed. After that the model was validated by solving Wu (2001) numerical example using the appropriate solution procedures which are simple to use algorithm and genetic algorithm. Then, a comparison made between the results of the above mentioned procedures and with Wu (2001) results. Also sensitivity analysis for developed model will be conducted and discussed to come out in conclusions. MATLAB software will be used to solve the numerical example. (see Figure 20).

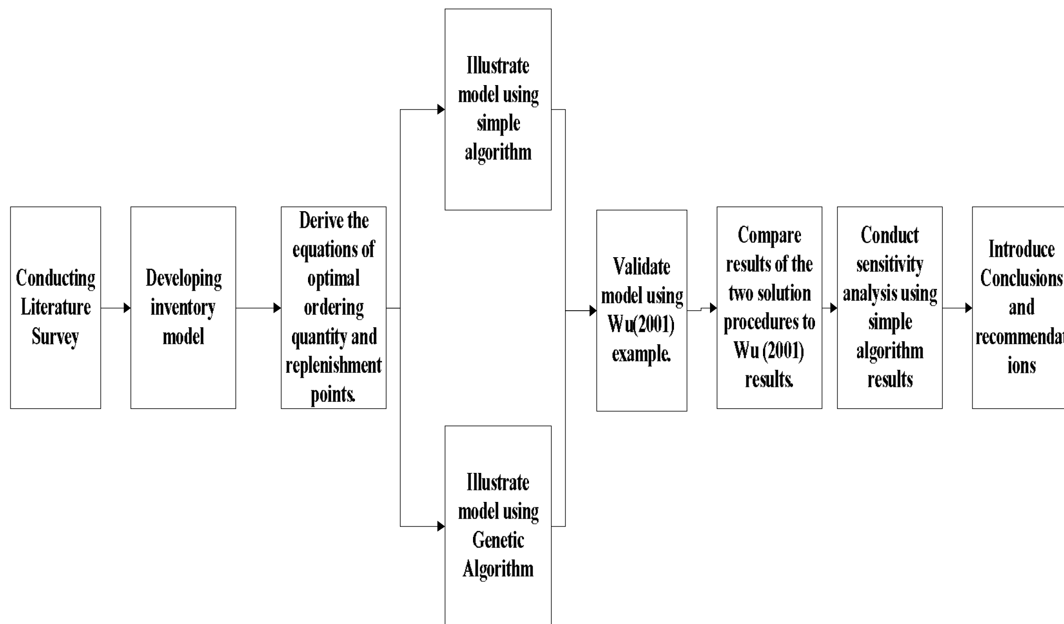


Figure 20: Methodology

Chapter 4: Model Formulation and Validation

In the stochastic demand, continuous review (Q, R) inventory model with backorders and permissible delay in payment an order of size Q is placed when the inventory level drops to a reorder point, R. The expected total annual cost of this model which developed by **Wu (2001)** is approximate to:

$$EAC(Q,R) = \frac{AD}{Q} + pD + \frac{hQ}{2} + (h + pI_c)(R - \mu) + \frac{\pi D}{Q} n(R) - \frac{D^2 pt_c^{\ 2} I_d}{2Q} - n(R) pt_c I_d \frac{D}{Q} + \frac{(Q - Dt_c)^2 pI_c}{2Q}$$

where A is the fixed order cost, D is the annual demand, p is the unit cost of item, h is the unit holding cost, μ is the mean lead-time demand, π is the unit shortage cost. x is the demand during the lead-time, F(x) the cumulative distribution function of the lead-time demand, and $n(R) = \int_R^{\infty 0} (x - R) f(x) dx$, Moreover, Q/2 is often called the cycle stock and (R -μ) is called the safety stock, he considered the effect of credit period on the above model. Let t_c be the credit period and h is redefined as the unit stock-holding cost per item per year excluding the interest charges for financing the stock. I_d is the rate at which the buyer from the sales amount derives interest and I_c be the interest rate applicable to the stock value after the credit period $I_c \geq I_d$). The holding cost for this stock is ($h+pI_c$) (R -μ), and the holding cost for the cycle stock is $hQ^2/2D$, lead time demand considered to be normally distributed, backorders are allowed and completely backlogged.

In our study, we extended Wu's (2001) model to include deteriorating inventory and back order cancellation in the light of credit period and the optimal order quantity and order point are determined.

The objective is to choose Q and R to minimize the average inventory holding, ordering and backorder costs, deteriorating cost and order cancellation cost (the cost resulted from lost customer good well).

We note that when a delay in payments is permitted, the buyers could reduce their total cost by paying later, and in this case, larger quantities should be ordered because of a credit period offered by the suppliers. Furthermore, from the buyers' point of view, in addition to getting some merits from suppliers, they may try other ways to improve the current system behaviors so as to minimize (maximize) the total cost (profit). In our study, we shall then consider that the option of investing in deteriorating cost and lost customer good well cost reduction is available. This model assumes that lead-time demand follows a normal distribution and that any demand, when out of stock, is backordered and that the backorder penalty is Proportional to the number of items backordered. these back orders may be backlogged completely, or partially because back ordered customers lack of patience so they may cancel their backorders before receiving them which affect the total cost for supplier to include lost sales cost and more over it may cost him to lost customer good well. Moreover, we assumed that the inventory in our model is deteriorating with deterioration rate constant with time, (See Figure 21).

Figure 21: Graphical representation for inventory system.

4.1 Notation

A: fixed ordering cost

r_c: interest rate applicable to the stock value after credit period

D: annual demand

r_d: the rate at which the buyer derive amount from the sales

Q: order quantity

n(R): Number of unit short; $n(R) = \int_R^{\infty} (x - R) f(x) dx$

p: unit cost of item

π:unit shortage cost

μ:mean lead time demand

h: unit holding cost

t_c: credit period

β: percent of lost sales

θ: deteriorate rate.

(R-μ) :safety stock.

c_g: cost of lost customer good well.

x: demand during lead time random variable.

k: safety stock factor.

4.2 Assumptions

1. Demand is constant over time.
2. Lead time demand is following a normal distribution with (μ,σ^2) and density function $f(D_L)$.
3. Shortages are permitted and partially backlogged.
4. Delay in payment is permitted.
5. Inventory items deteriorate with constant rate θ.

4.3 Formulation

Suppose the lead-time demand, X, follows a normal distribution with mean μ and standard deviation σ. Also, consider that the reorder point, R=expected demand during lead time + safety stock (SS), and SS = k ,i.e., R=μ+ kσ, where k(≥0) is the safety factor thus the expected number of units short per cycle is n(R), $n(R)=\int_R^\infty (x-R)f(x)dx$ By substituting R=μ+ kσ and z = (x -μ)/σ where x=σz+μ to standardize the random variable x, then:

$$\int_R^\infty (x-R)f(x)dx=\int_k^\infty (\sigma z+\mu)-(\mu+\sigma k)\phi(z)dz=\sigma\int_k^\infty (z-k)\phi(z)dz$$

And $\int_{k}^{\infty}(z-k)\phi(z)dz = \int_{k}^{\infty} z\phi(z)dz - \int_{k}^{\infty} k\phi(z)dz = \phi(k) - k[1-\Phi(k)]$

Then $n(R) = \int_{R}^{\infty}(x-R)f(x)dx = \sigma\{\phi(k) - k[1-\Phi(k)]\}$.............(1)

Where ϕ and Φ denote the standard normal pdf and cdf (probability and cumulative distribution function), respectively. To make equation more simple we assumed that $\Psi(k) = \phi(k) - k[1-\Phi(k)]$...........(2)

This means that $n(R) = \sigma\Psi(k)$.............(3)

4.3.1 Total Annual Cost Function TAC

Total annual cost for our inventory includes the ordering cost, cycle inventory cost, safety stock cost, shortage cost lost sales cost, interest charged for on hand inventory and deterioration cost. Also we excluded from the TAC interest earned from demand sales during credit period, interest earned from sales of back orders from previous period as follows:

- Ordering cost $= \frac{AD}{Q} + pD$
- Cycle inventory cost $= \frac{hQ}{2}$
- Interest charged for On hand inventory at tc $= \frac{(Q - Dt_c - \theta t_c)^2 \, pr_c}{2Q}$
- Safety stock cost $= (h + pr_c)(R - \mu - \theta t_c)$
- Shortage cost $= \frac{\pi \, n(R) D}{Q}$
- Lost sales cost $= \frac{n(R)\,\beta\,(p + c_g)D}{Q}$
- Interest earned from demand sales during credit period $= \frac{-D^2 p t_c^{\,2} r_d}{2Q}$
- Interest earned from sales of backorders from previous period $= \frac{-n(R)\, p t_c r_d D}{Q}$
- Deterioration cost $= \frac{P\,\theta\, t_c\, D}{Q}$

So the Total Annual Cost TAC function is:

$$TAC(Q,R)=\frac{AD}{Q}+pD+\frac{hQ}{2}+\frac{Q-Dt_c-\theta t_c 2\,pr_c}{2Q}+(h+pr_c)(R-\mu-\theta t_c)+\frac{\pi\, n(R)D}{Q}$$
$$+\frac{n(R)\,\beta\,(p+c_g)D}{Q}-\frac{D^2pt_c{}^2r_d}{2Q}-\frac{n(R)\,pt_cr_dD}{Q}+\frac{P\,\theta\,t_c\,D}{Q}\text{.....................(4)}$$

And by substituting equation (3) in equation (4) we find that:

$$TAC(Q,k)=\frac{AD}{Q}+pD+\frac{hQ}{2}+\frac{Q-Dt_c-\theta t_c 2\,pr_c}{2Q}+(h+pr_c)(R-\mu-\theta t_c)+\frac{\pi\,\sigma\Psi(k)D}{Q}$$
$$+\frac{\sigma\Psi(k)\,\beta\,(p+c_g)D}{Q}-\frac{D^2pt_c{}^2r_d}{2Q}-\frac{\sigma\Psi(k)\,pt_cr_dD}{Q}+\frac{P\,\theta\,t_c\,D}{Q}\text{.....................(5)}$$

In order to find the optimum order Quantity (Q), and optimum reordering point (R), we find the first derivative of the TAC function with respect to Q then with respect to k, then we solved the system of equations using the developed algorithm and the Genetic algorithms.

4.3.2 First derivative of TAC with respect to Q:

To find the equation representing Q we will first find the derivative of equation (5) then we will make the derivative equal to zero.

In order to find the derivative of the total annual cost with respect to order quantity Q, the elements of the TAC (Q, R) was derived with respect to Q as following:

$$\frac{\partial TAC}{\partial Q}=\frac{-AD}{Q^2}+\frac{h}{2}+\frac{pr_c}{2}-\frac{(D-\theta)^2\,pr_c\,t_c{}^2}{2Q^2}+-\frac{\pi\sigma\Psi(k)D}{Q^2}+-\frac{\sigma\Psi(k)\,\beta\,(p+c_g)D}{Q^2}$$
$$\frac{D^2pt_c{}^2r_d}{2Q^2}+\frac{\sigma\Psi(k)\,pt_cr_dD}{Q^2}+\frac{P\,\theta\,t_c\,D}{Q^2}\text{……….(6)}$$

And by making equation (6) equal to zero we find that:

$$Q=\sqrt{\frac{AD+\frac{(D-\theta)^2\,pr_c\,t_c{}^2}{2}+\pi\sigma\Psi(k)D+\sigma\Psi(k)\beta(p+c_g)D+\frac{p\theta t_cD-D^2pt_c{}^2r_d}{2}-\sigma\Psi(k)pt_cr_dD}{(h+pr_c)/2}}\text{…..(7)}$$

4.3.3 First derivative of TAC with respect to k:

To find the equation representing k we find the derivative of equation (4) with respect to k, and then we will make the derivative equal to zero.

In order to find the derivative of the total annual cost with respect to replenishment safety factor, k, the elements of the TAC (Q, R) was derived with respect to k as following:

$$\frac{\partial TAC}{\partial k} = (h + prc) + \frac{\pi D(\Phi(K) - 1)}{Q} + \frac{\beta (p + c_g) D(\Phi(K) - 1)}{Q} + \frac{pt_c r_d D(\Phi(K) - 1)}{Q} \ldots\ldots(8)$$

And by making equation (8) equal to zero the result is:

$$(1 - \Phi(k)) = \frac{Q(h + pr_c)}{\pi D + \beta (p + c_g) D + pt_c r_d D} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

4.3.4 Simple to Use Algorithm

First we developed a simple-to-use algorithm to validate the above mentioned formulation, by setting initial condition for order quantity Q_0 to be equal to the Economic Order Quantity EOQ= $\sqrt{\frac{2AD}{h}}$, and use equation (8) to find value of k_i for the initial condition, then using the value of k in equation (6) we find the next value of Q_{i+1}, the termination criteria for this algorithm is when Q_i=Q_{i-1} and k_i=k_{i-1}. this algorithm is right under the conditions of order Quantity Q is positive and safety factor k is positive too.

<u>The simple to use algorithm will be:</u>

<u>Step 0:</u> set EOQ= $\sqrt{\frac{2AD}{h}}$ as an estimate of Q, EOQ=Q_0,i=0

<u>Step1:</u> use equation 9 with Q_i=Q_o to find k, k=k_i,

<u>Step2:</u> use equation 7 with $k=k_i$ to find Q_{i+1} if $Q_i=Q_{i-1}$ and $k_i=k_{i-1}$ then stop otherwise set $i=i+1$, go to step1, $Q>0$, $k\geq 0$.

Step 3: fined R and TAC (Q, k) for the optimal values of k and Q .where $R=\mu+ k\sigma$.

4.3.5 Genetic Algorithm

In our problem we used the GA tool in the MATLAB program to find the minimum total annual cost for a continuous review inventory model, and to find the order quantity and replenishment point associated with this value, see appendix 2. Objective function is to minimize the total cost function (equation 4) such that $Q\geq 0$ and $k\geq 0$.

$$TAC(Q,k) = \frac{AD}{Q} + pD + \frac{hQ}{2} + \frac{Q - Dt_c - \theta t_c 2\, pr_c}{2Q} + (h + pr_c)(R - \mu - \theta t_c) + \frac{\pi\, \sigma\Psi(k)D}{Q}$$
$$+ \frac{\sigma\Psi(k)\, \beta\, (p + c_g)D}{Q} - \frac{D^2 p t_c{}^2 r_d}{2Q} - \frac{\sigma\Psi(k)\, p t_c r_d D}{Q} + \frac{P\, \theta\, t_c\, D}{Q} \ldots\ldots\ldots\ldots(4)$$

4.3.6 Model validation

In order to validate the above formulated model we used the same example1 mentioned in Wu (2001), using the values of θ and β to be 0.03, 0.03 respectively.

$A=50$, $D=200$, $p=10$, $h=2$, $\pi=5$, $t_c=0.1$, $r_c=0.15$, $r_d=0.12$, $\mu=50$, $\sigma=9$, $c_g=6$,□=0.03, $\theta=0.03$.

1. Using MATLAB the simple to use algorithm was validated and the optimum order quantity and replenishment point (Q, R) were (81.1575, 55.9686) respectively. (See appendix 2).

2. Using GA by MATLAB the same model was validated (see appendix 3) and the optimum order quantity and replenishment point (Q, R) were as in table 1, where the

first variable is equal to Q which is 82 and the second variable is k, the safety stock, with 0.293 from which we find R to equal 52.277.

Table 1: Comparison between the results of both (simple to use algorithm and GA).

Optimal value for variables	**Simple to use algorithm**	**GA**	**Wu(2001) model results**
Q	81.1575	82	82
R	55.9686	52.277	55
TAC	2275.0	2275.0041	2273.9

The results of using our developed algorithm and genetic algorithm to solve Wu (2001) model are as shown in table 2.

Table 2: Results of Wu (2001) model using our developed simple to use algorithm.

Optimal value for variables	**Wu (2001) model using our developed simple to use algorithm**
Q	81.3035
R	55.3023
TAC	2273.1

Chapter 5: sensitivity analysis

After the continuous review inventory model for deteriorating items with delay in payment and partial backlogging was validated using the simple-to-use algorithm and GA, sensitivity analysis was conducted to present the effect of changing parameters on the (Q,R) values and on the TAC value.

The sensitivity analysis was performed by changing each of the parameters (A, D, p, h, π, t_c, r_d, r_c, μ and σ) by +50%;+25%; -25% and -50% taking one parameter at a time while keeping remaining unchanged, as shown in Table3.

And for variables (c_g, β and θ) the sensitivity analysis was conducted as shown in Table 4, Table 5 and Table 6 respectively.

Table 3: Effect of changing parameters on the values of Q, R, and TAC.

variable	percent change%	value	Q	R	TCU	Percentage of change in		
						Q	R	TCU
A	50	75.00	98.4470	54.5230	2330.4	0.1729	-0.0145	0.5550
	25	62.50	90.2347	55.1928	2304.0	0.0908	-0.0078	0.2910
	0	50.00	81.1578	55.9686	2274.9			
	-25	37.50	70.8638	56.9061	2242.2	-0.1029	0.0094	-0.3270
	-50	25.00	58.6712	58.1246	2203.8	-0.2249	0.0216	-0.7110
D	50	300.00	97.9587	57.4443	3323.9	0.1680	0.0148	10.4900
	25	250.00	89.9512	56.8025	2801.1	0.0879	0.0083	5.2620
	0	200.00	81.1578	55.9686	2274.9			
	-25	150.00	71.2842	54.7975	1743.8	-0.0987	-0.0117	-5.3110
	-50	100.00	59.8206	52.8947	1204.5	-0.2134	-0.0307	-10.7040
p	50	15.00	74.2764	55.4704	3293.9	-0.0688	-0.0050	10.1900
	25	12.50	77.4675	55.7049	2784.8	-0.0369	-0.0026	5.0990
	0	10.00	81.1578	55.9686	2274.9			
	-25	7.50	85.4924	56.2676	1764.2	0.0433	0.0030	-5.1070
	-50	5.00	90.6862	56.6100	1252.6	0.0953	0.0064	-10.2230
h	50	3.00	72.4932	54.9477	2318.5	-0.0866	-0.0102	0.4360
	25	2.50	76.4078	55.4373	2297.4	-0.0475	-0.0053	0.2250
	0	2.00	81.1578	55.9686	2274.9			
	-25	1.50	87.0774	56.5540	2250.9	0.0592	0.0059	-0.2400
	-50	1.00	94.7281	57.2128	2224.8	0.1357	0.0124	-0.5010
π	50	7.50	80.6612	58.4370	2281.8	-0.0050	0.0247	0.0690

variable	percent change%	value	Q	R	TCU	Percentage of change in		
						Q	R	TCU
	25	6.25	80.8688	57.3647	2278.8	-0.0029	0.0140	0.0390
	0	5.00	81.1578	55.9686	2274.9			
	-25	3.75	81.6035	54.0022	2269.6	0.0045	-0.0197	-0.0530
	-50	2.50	82.4563	50.7799	2261.3	0.0130	-0.0519	-0.1360
t_c	50	0.15	81.3038	56.0316	2260.7	0.0015	0.0006	-0.1420
	25	0.13	81.3121	56.0338	2260.1	0.0015	0.0007	-0.1480
	0	0.10	81.1578	55.9686	2274.9			
	-25	0.08	81.1286	55.9328	2282.2	-0.0003	-0.0004	0.0730
	-50	0.05	81.1295	55.8941	2289.6	-0.0003	-0.0007	0.1470
r_c	50	0.23	74.8850	55.1351	2295.1	-0.0627	-0.0083	0.2020
	25	0.19	77.7804	55.5400	2285.4	-0.0338	-0.0043	0.1050
	0	0.15	81.1578	55.9686	2274.9			
	-25	0.11	85.1575	56.4264	2263.7	0.0400	0.0046	-0.1120
	-50	0.08	89.9876	56.9210	2251.5	0.0883	0.0095	-0.2340
r_d	50	0.18	80.5544	56.0972	2273.3	-0.0060	0.0013	-0.0160
	25	0.15	80.8557	56.0331	2274.1	-0.0030	0.0006	-0.0080
	0	0.12	81.1578	55.9686	2274.9			
	-25	0.09	81.4598	55.9037	2275.8	0.0030	-0.0006	0.0090
	-50	0.06	81.7627	55.8383	2276.6	0.0060	-0.0013	0.0170
σ	50	13.50	84.0559	58.5743	2294.2	0.0290	0.0261	0.1930
	25	11.25	82.5860	57.3050	2284.6	0.0143	0.0134	0.0970
	0	9.00	81.1578	55.9686	2274.9			
	-25	6.75	79.7678	54.5682	2265.2	-0.0139	-0.0140	-0.0970
	-50	4.50	78.4153	53.1059	2255.3	-0.0274	-0.0286	-0.1960
c_g	50	9.00	81.1328	56.0840	2275.3	-0.0002	0.0012	0.0040
	25	7.50	81.1450	56.0266	2275.1	-0.0001	0.0006	0.0020
	0	6.00	81.1578	55.9686	2274.9			
	-25	4.50	81.1701	55.9099	2274.8	0.0001	-0.0006	-0.0010
	-50	3.00	81.1829	55.8504	2274.6	0.0003	-0.0012	-0.0030
β	50	0.05	81.0932	56.2703	2275.8	-0.0006	0.0030	0.0090
	25	0.04	81.1247	56.1218	2275.4	-0.0003	0.0015	0.0050
	0	0.03	81.1578	55.9686	2274.9			
	-25	0.02	81.1916	55.8104	2274.5	0.0003	-0.0016	-0.0040
	-50	0.02	81.2271	55.6468	2274.1	0.0007	-0.0032	-0.0080

Table 4: Effect of changing Lost customer good well cost on the values of Q, R, and TAC.

cg	TAC	Q	R
1	2274.5	81.2003	55.77
2	2274.6	81.1916	55.8104
3	2274.7	81.1829	55.8504
4	2274.8	81.1743	55.8901

cg	TAC	Q	R
5	2274.9	81.1659	55.9295
6	2275	81.1575	55.9686
7	2275.1	81.1492	56.0074
8	2275.2	81.1409	56.0458
9	2275.3	81.1328	56.084
10	2275.4	81.1247	56.1218

Table 5: Effect of changing cancellation rate on the values of Q, R, and TAC.

β	TAC	Q	R
0.01	2273.8	81.2517	55.5346
0.02	2274.4	81.2033	55.756
0.03	2275	81.1575	55.9686
0.04	2275.6	81.1191	56.1718
0.05	2276.1	81.0729	56.3668
0.06	2276.6	81.0337	56.5542
0.07	2277.1	80.9964	56.7345
0.08	2277.6	80.9608	56.9081
0.09	2278.1	80.9268	57.0756
0.1	2278.5	80.8943	57.2373

Table 6: Effect of changing Deterioration rate on the values of Q, R, and TAC.

θ	TAC	Q	R
0.01	2.275	81.1577	55.9686
0.02	2.275	81.1576	55.9686
0.03	2.275	81.1575	55.9686
0.04	2.275	81.1573	55.9686
0.05	2.275	81.1572	55.9686
0.06	2.2751	81.1571	55.9686
0.07	2.2751	81.157	55.9686
0.08	2.2751	81.1569	55.9686
0.09	2.2751	81.1568	55.9686
0.1	2.2751	81.1567	55.9687
0.15	2.2752	81.1561	55.9687
0.2	2.2753	81.1555	55.9688
0.25	2.2754	81.1549	55.9688
0.3	2.2755	81.1544	55.9689
0.35	2.2756	81.1538	55.9689
0.4	2.2757	81.1532	55.969
0.45	2.2758	81.1526	55.969
0.5	2.2759	81.1521	55.9691
0.55	2.276	81.1515	55.9691
1	2.2769	81.1463	55.9696

Based on Table 3, we can observe the following:

1. Q and TAC increases when A is increasing, but on almost R has a slight change in value with changing A. (see Figure 22 and Figure 23).

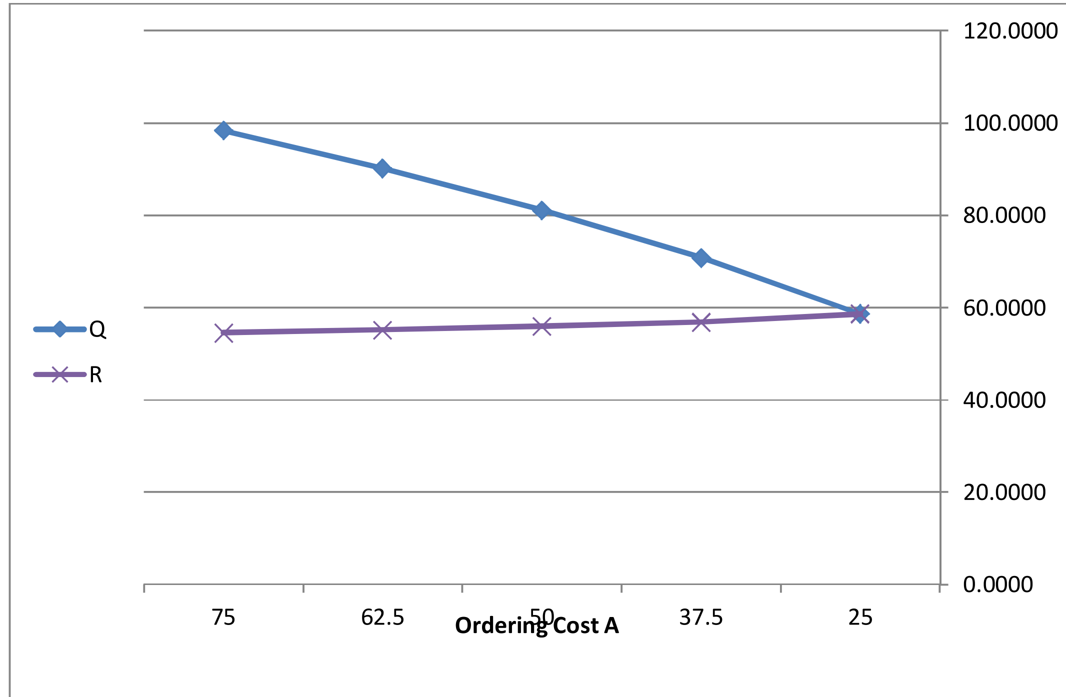


Figure 22: Effect of changing in the ordering cost on the values of Q and R.

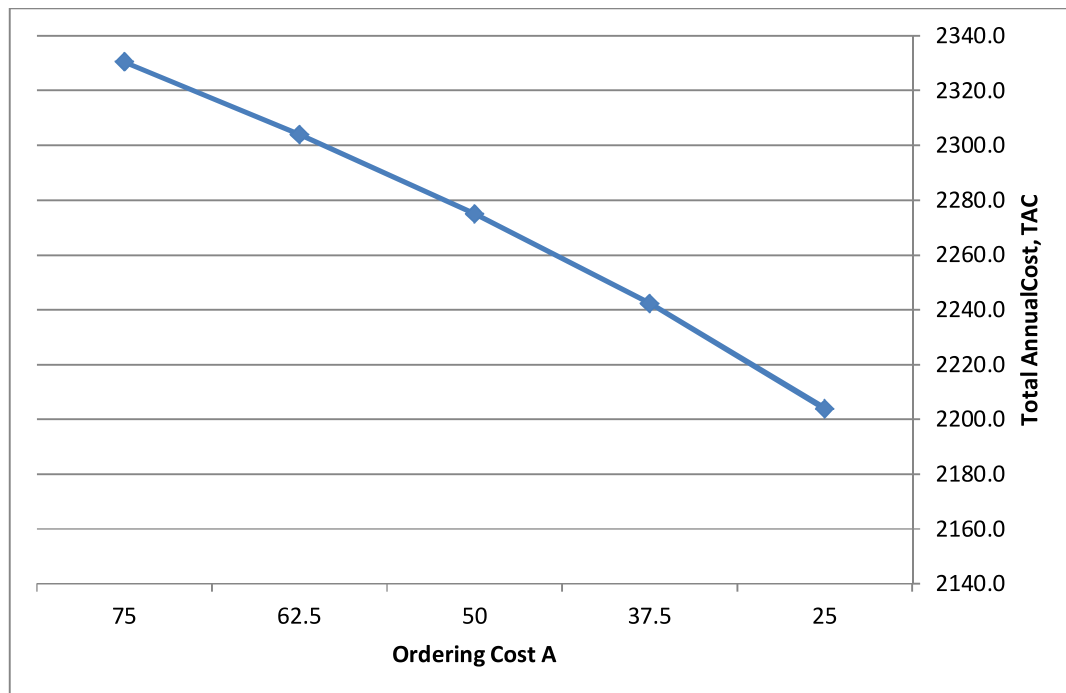


Figure 23: Effect of changing in the ordering cost on the value of TCA.

2. Q and TAC increase with increasing annual demand, and R has a slight increase by increasing demand, as you can see in Figures 24 and 25.

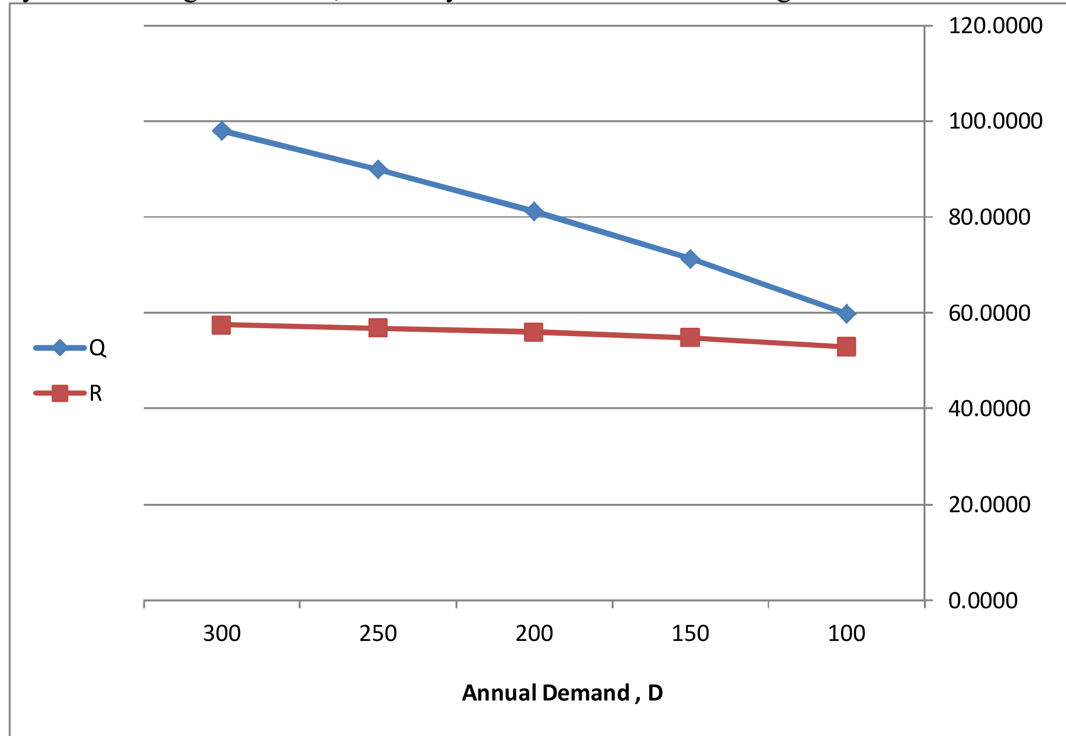


Figure 24: Effect of changing in the Annual Demand on the value of Q and R.

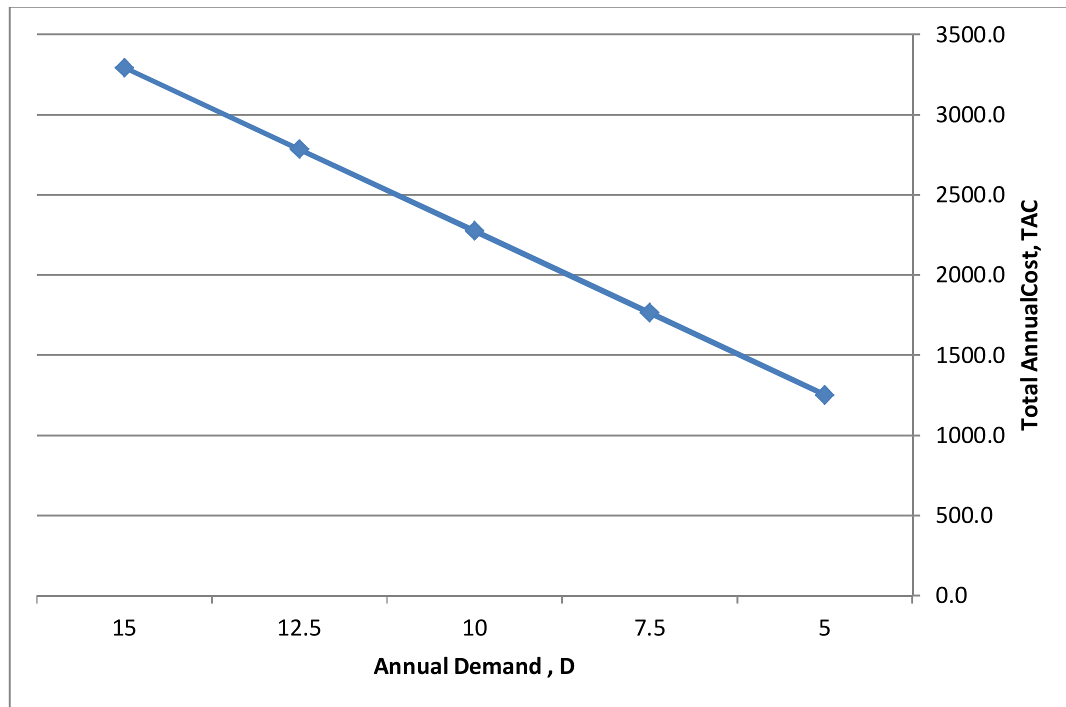


Figure 25: Effect of changing in the Annual Demand on the value of TAC.

3. As in Figure 26, Q decreases with increasing unit purchase cost p, TAC increases by increasing p (see Figure 27), and R has no change when increasing p.

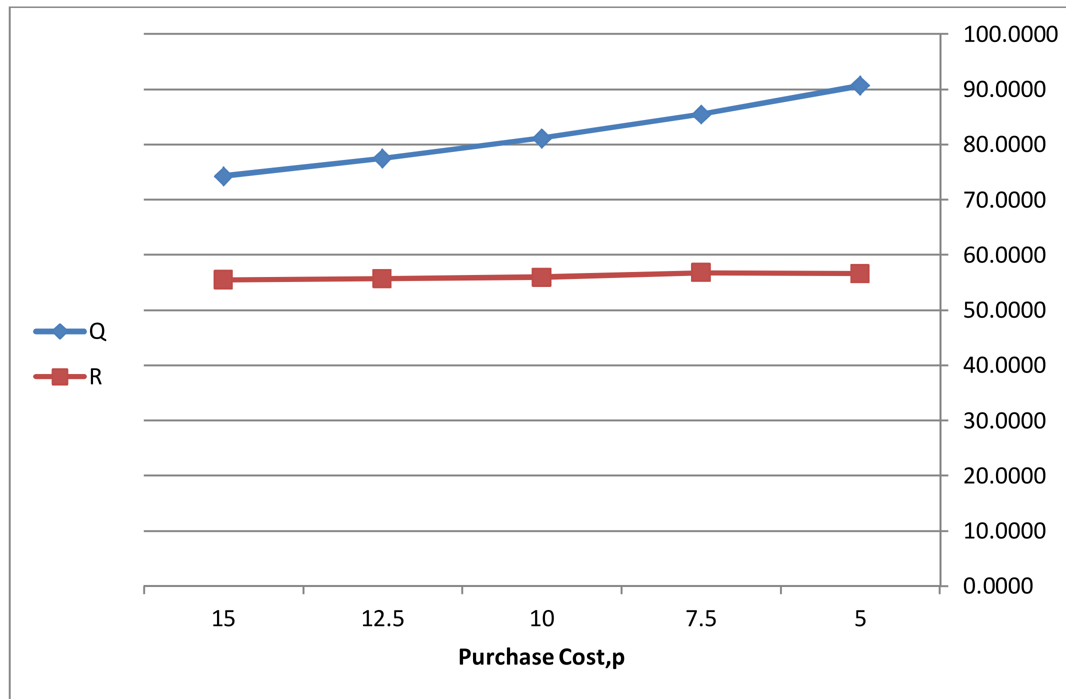


Figure 26: Effect of changing in the unit purchase cost on the value of Q and R.

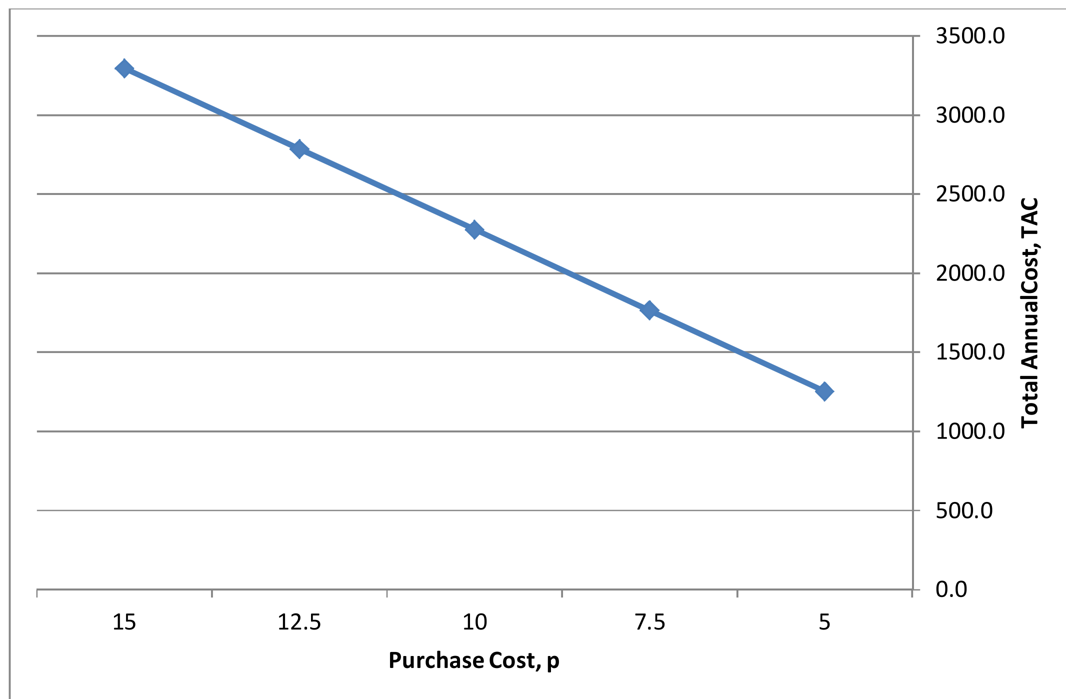


Figure 27: Effect of changing in the unit purchase cost on the value of TAC.

4. As in Figure 28, Q decreases with increasing unit holding cost h, R has no change when increasing, and TAC increases by increasing h see Figure 29.

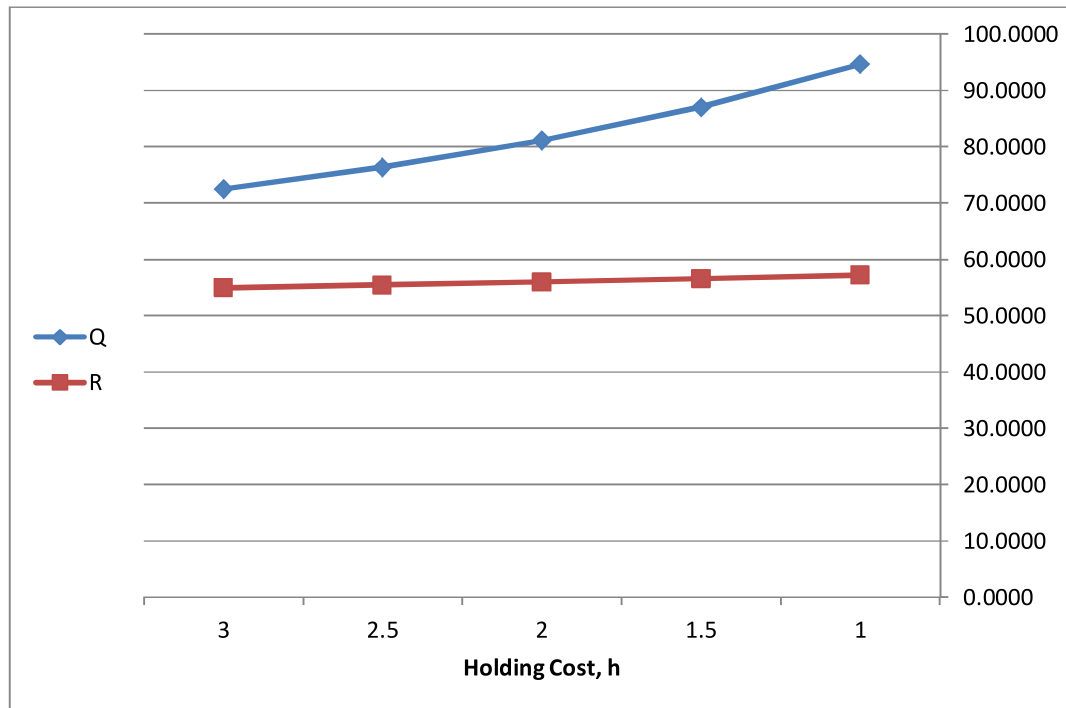


Figure 28: Effect of changing in the holding cost on the value of Q and R.

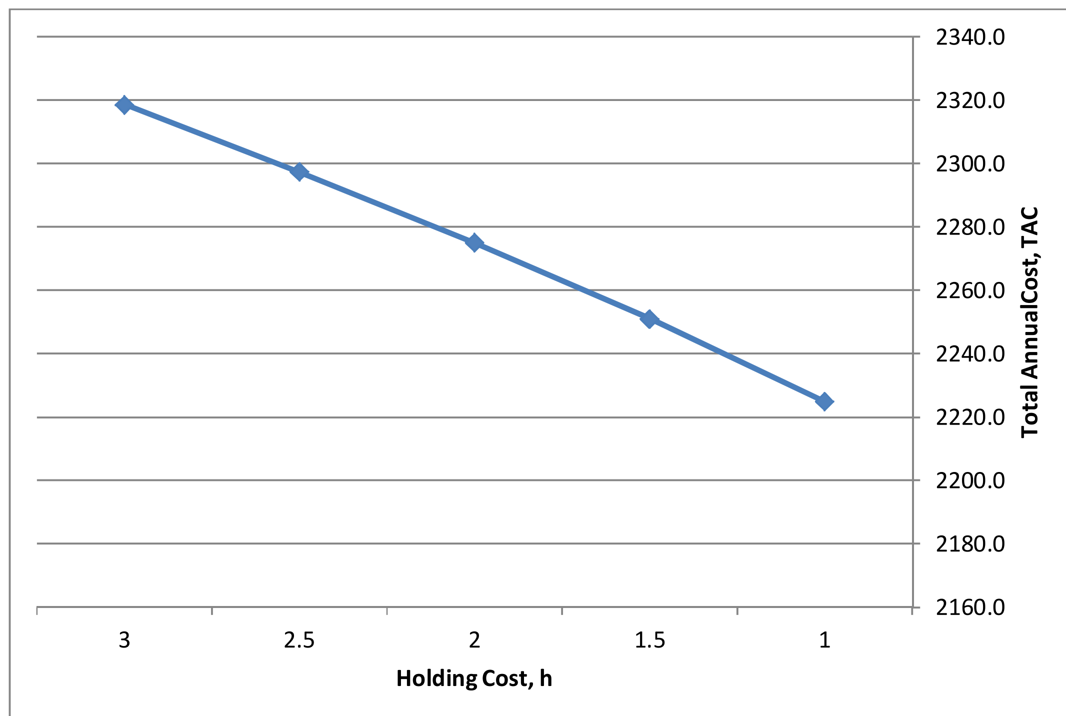


Figure 29: Effect of changing in the holding cost on the value of TAC.

5. When increasing the shortage cost Q decreases slightly and R increasing slightly but TAC has a large increase in value as in Figures 30 and 31.

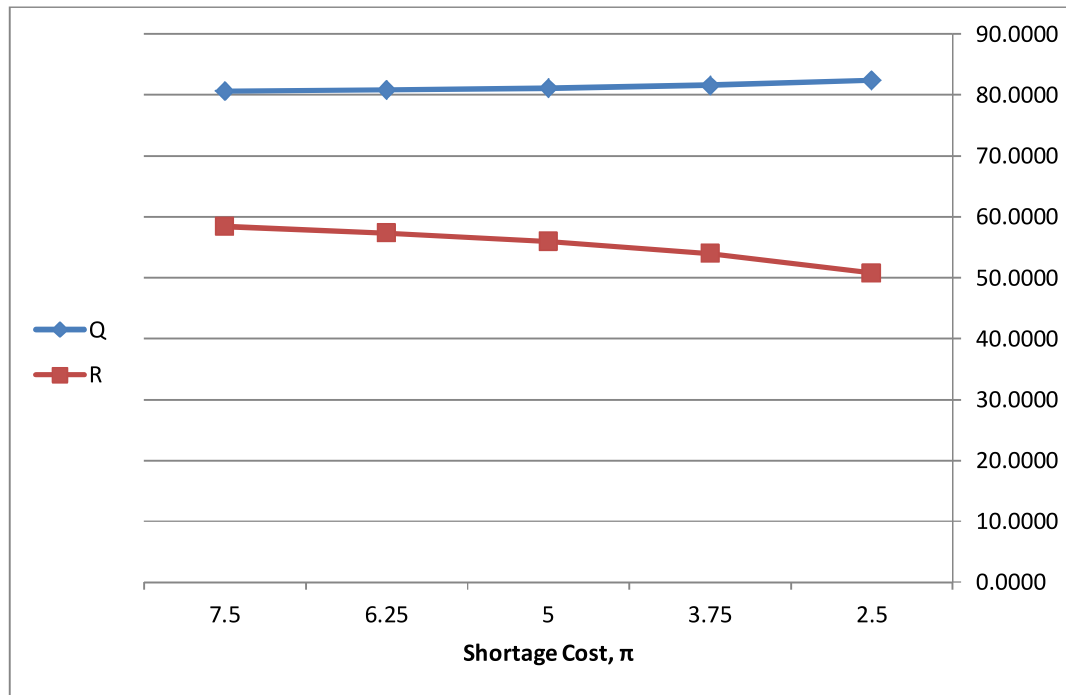


Figure 30: Effect of changing in the shortage cost on the value of Q and R.

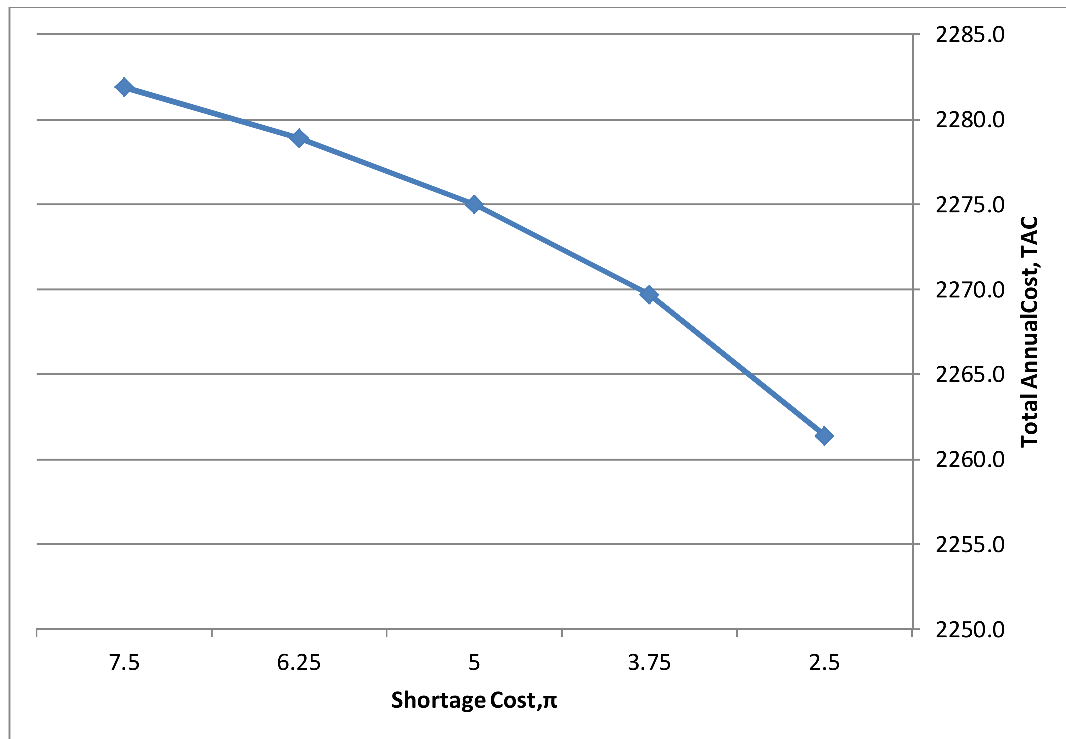


Figure 31: Effect of changing in the shortage cost on the value of TAC.

6. When increasing the delay period Q and R has no change in value but the TAC will increase in a large value as in Figures 32 and 33.

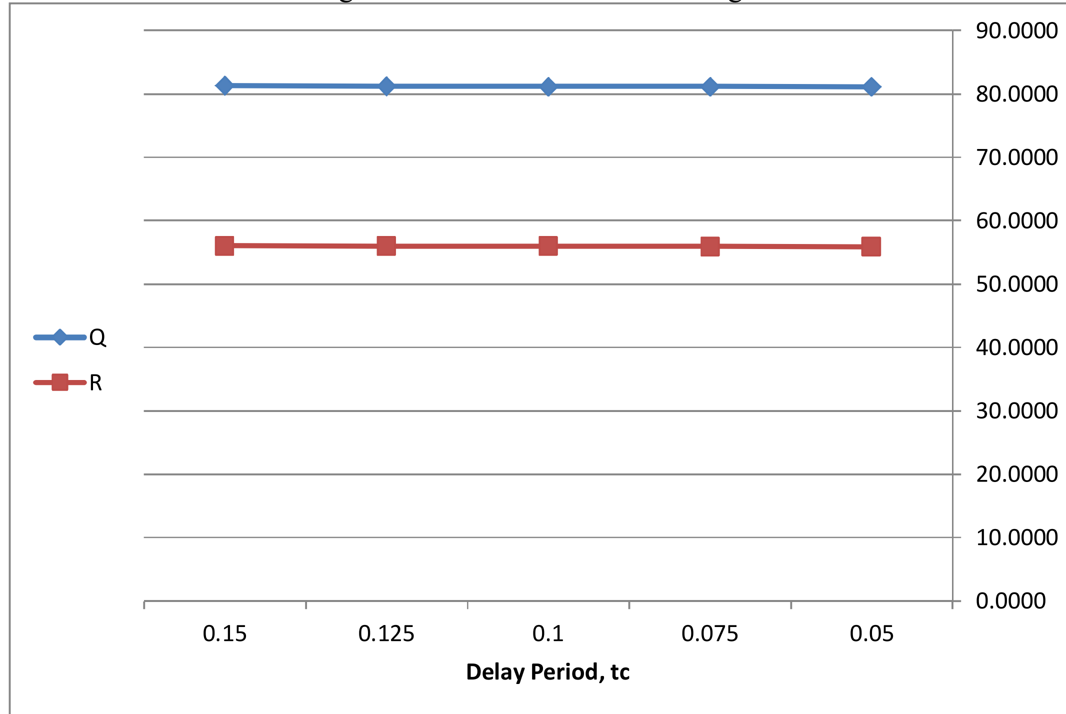


Figure 32: Effect of changing in the delay period on the value of Q and R.

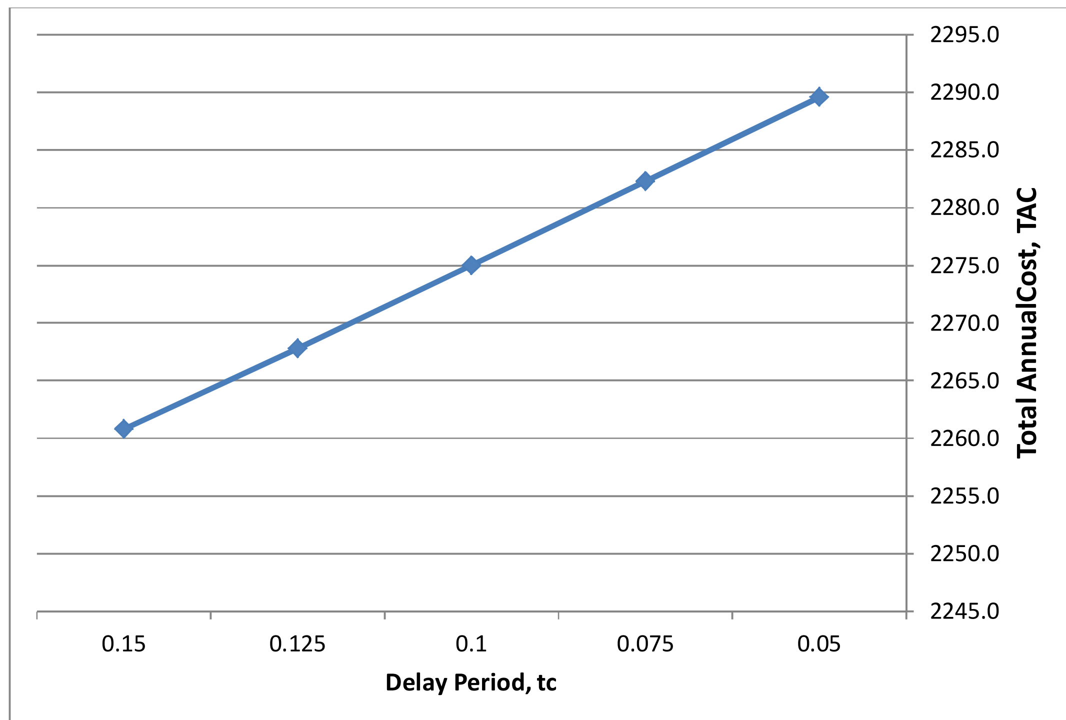


Figure 33: Effect of changing in the delay period on the value of TAC.

7. As in Figure 34, we can observe a slight decrease in the value of Q and the value of R when the value of interest rate charged on the on hand inventory after delay period is increased. And as you can see in Figure 35, TAC will increase sharply if the value increased.

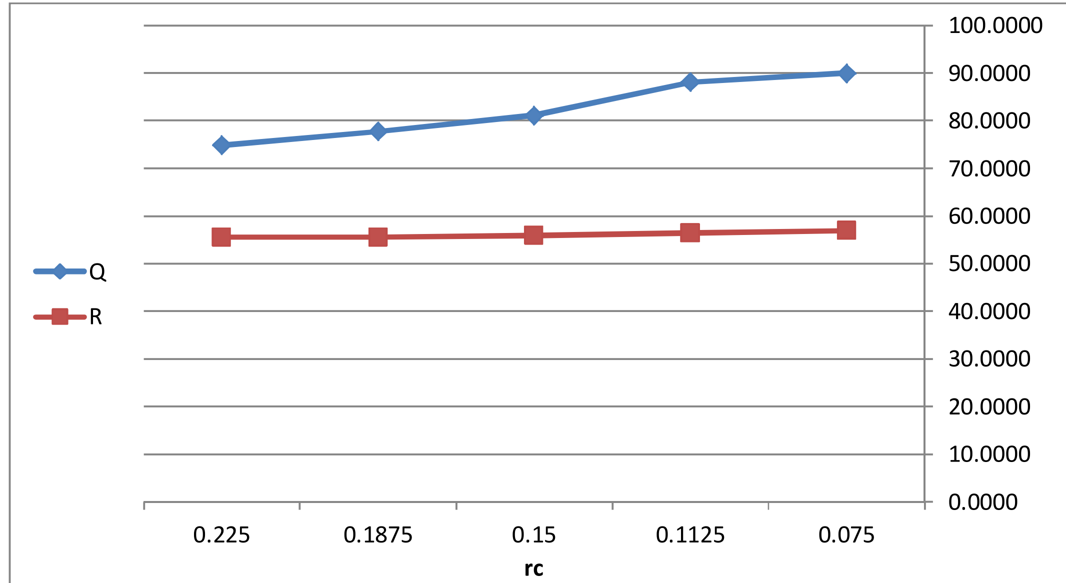


Figure 34: Effect of changing in the interest rate charged on the on hand inventory after delay period on the value of Q and R.

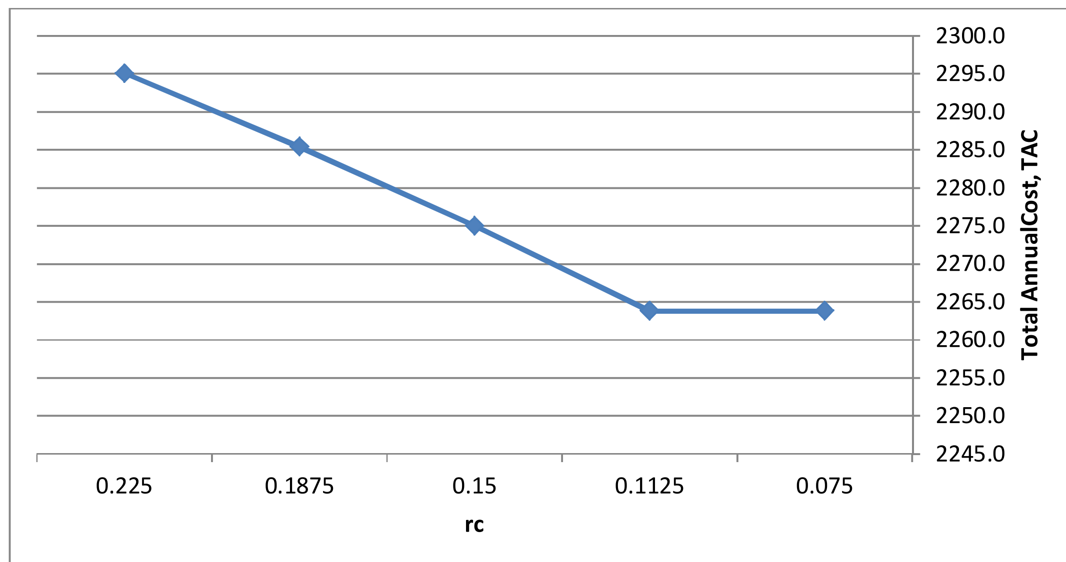


Figure 35: Effect of changing in the interest rate charged on the on hand inventory after delay period on the value of TAC.

8. R will not change by increasing the interest rate derived from the amount of sales rd during the credit period, Q will has a slight decrease, but TAC will decrease sharply by increasing rd. As in Figures 36 and 37.

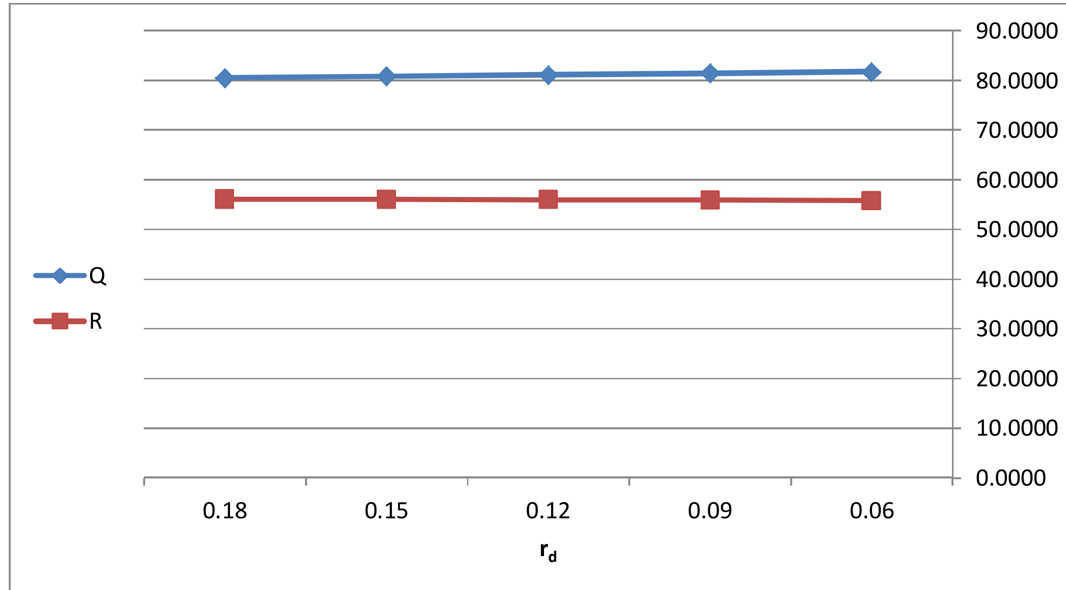


Figure 36: Effect of changing in the 'interest rate during delay period' on the value of Q and R.

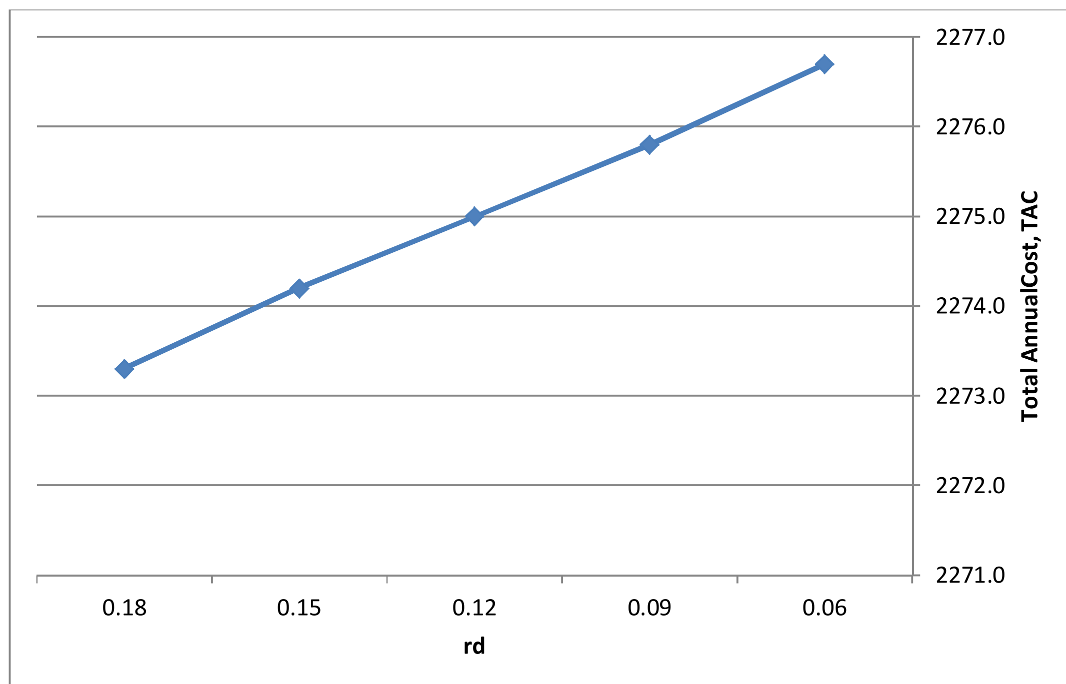


Figure 37: Effect of changing in the 'interest rate during delay period' on the value of TAC.

9. When the mean of lead time demand μ increases, R will increase, but Q and TAC will not change by changing μ, (As in Figures 38 and 39).

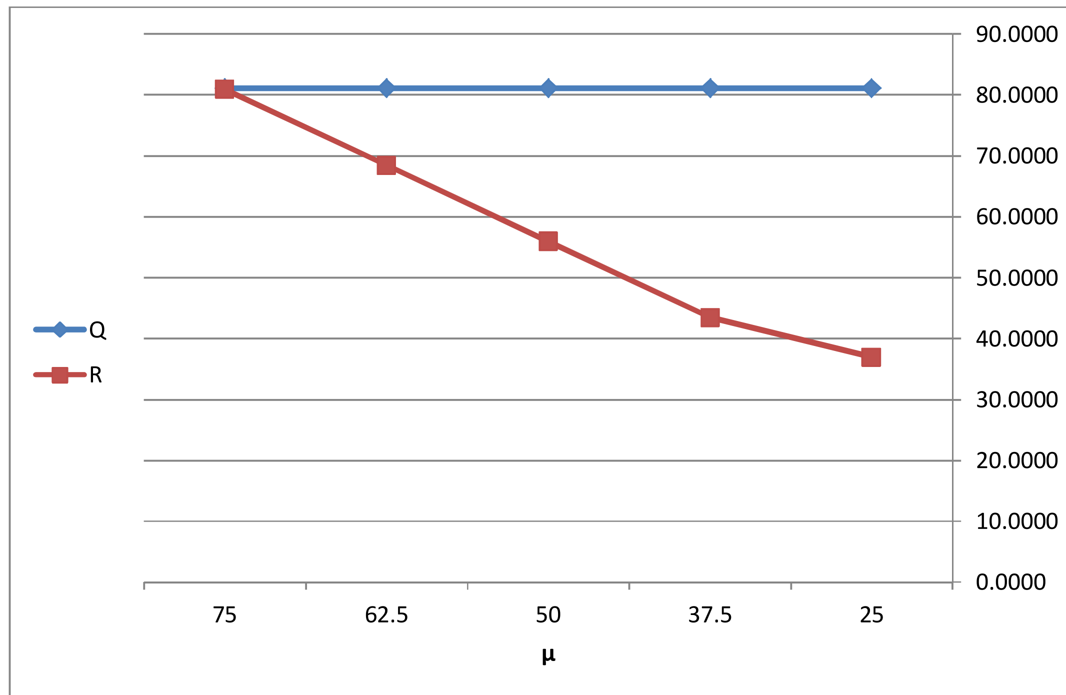


Figure 38: Effect of changing in the mean-lead time demand on the value of Q and R.

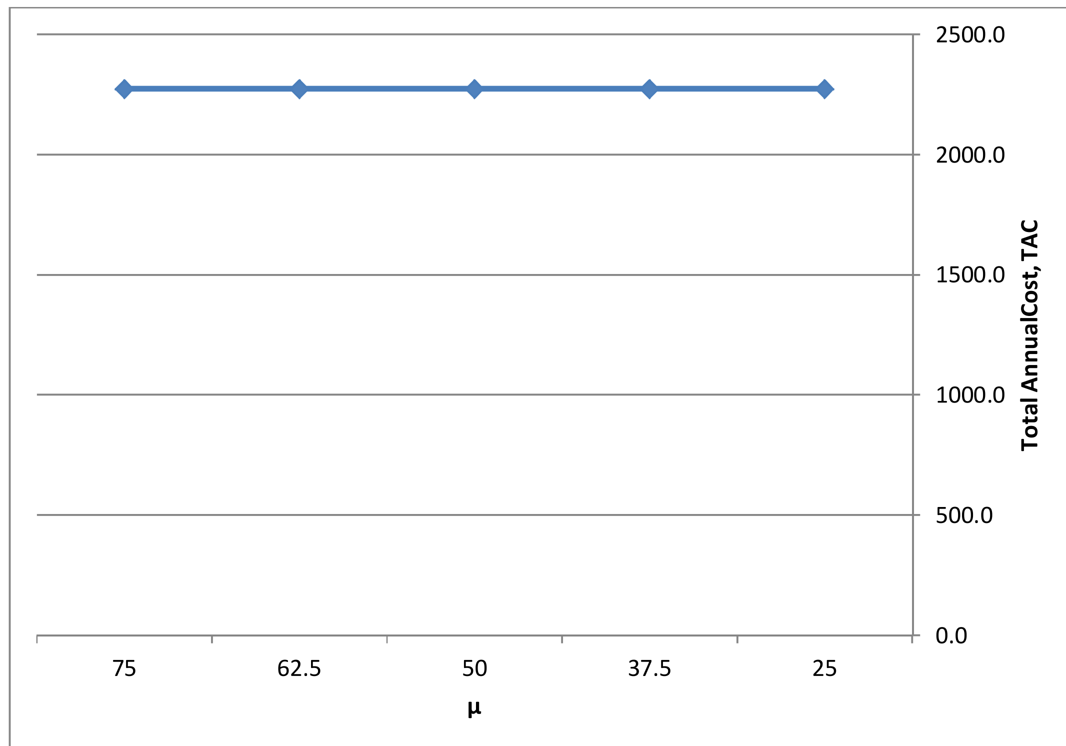


Figure 39: Effect of changing in the mean-lead time demand on the value of TAC.

10. When the standard deviation of lead time demand increases the values of Q and R will increase slightly but TAC is highly sensitive to increase in σ. (see Figure 40 and Figure 41).

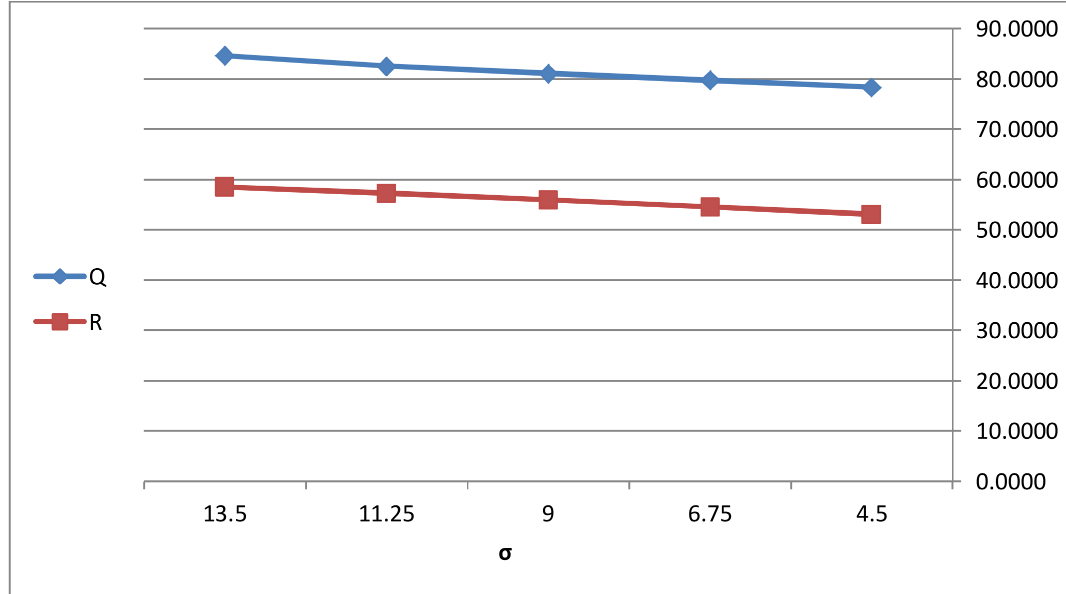


Figure 40: Effect of changing in the standard deviation of lead time demand on the value of Q and R.

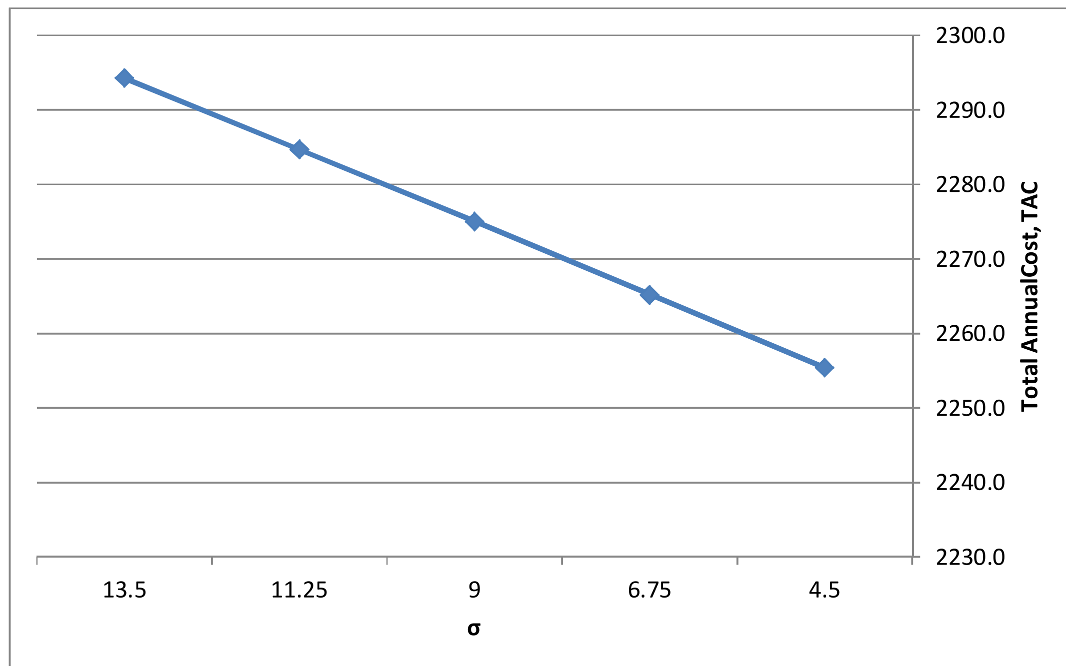


Figure 41: Effect of changing in the standard deviation of lead time demand on the value of TAC.

Based on Table 4, Table 5 and Table 6 we can obtain the following:

1. When the cost of lost customer good well, deterioration rate and cancellation rate increase Q and R will be slight sensitive to this increase, (see Figure 42, 43, 45, 46, 48 and 49).

2. But When the cost of lost customer good well and cancellation rate increase TAC will increase as shown in Figures 44 and 47. But when deterioration rate increase TAC will be less sensitive to this change. As shown in figure 50.



Figure 42: Effect of changing in the cost of lost customer good well on the value of Q.



Figure 43: Effect of changing in the cost of lost customer good well on the value of R.



Figure 44: Effect of changing in the cost of lost customer good well on the value of TAC.

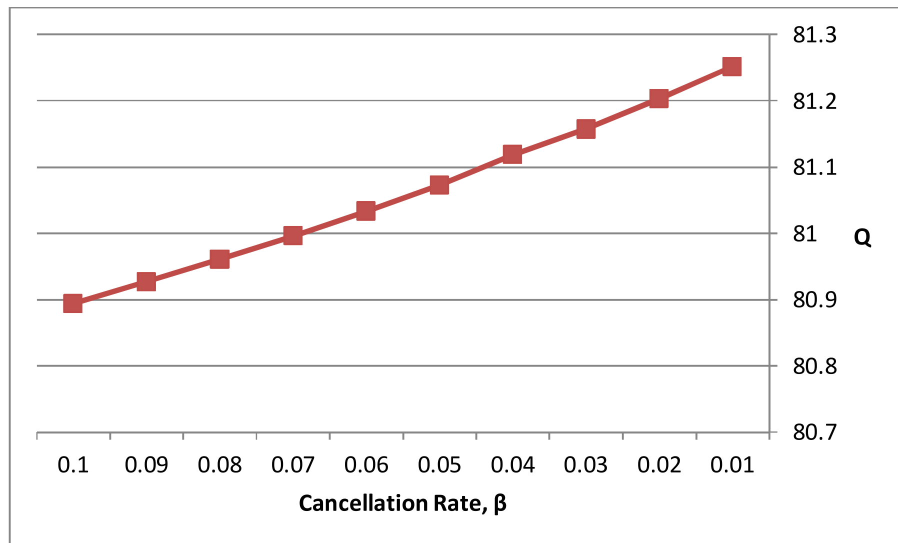


Figure 45: Effect of changing in cancellation rate on the value of Q.

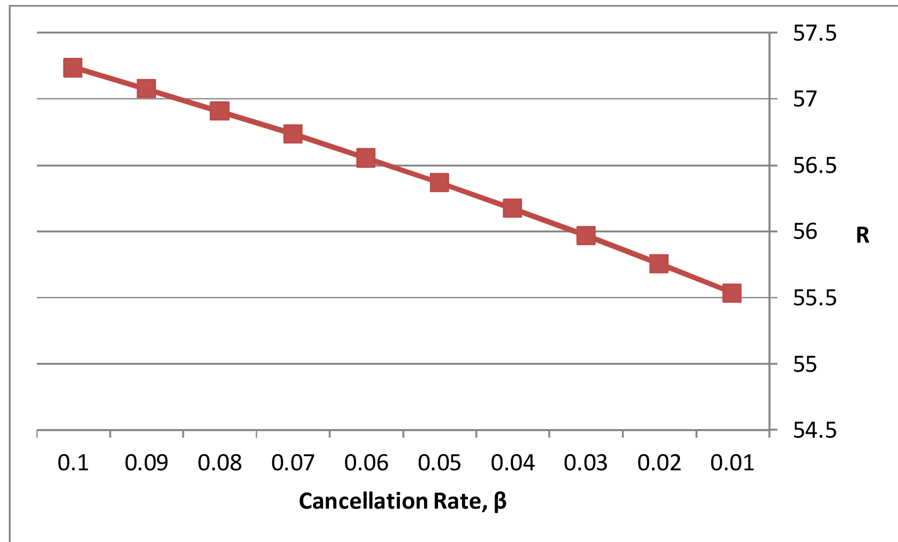


Figure 46: Effect of changing in cancellation rate on the value of R.

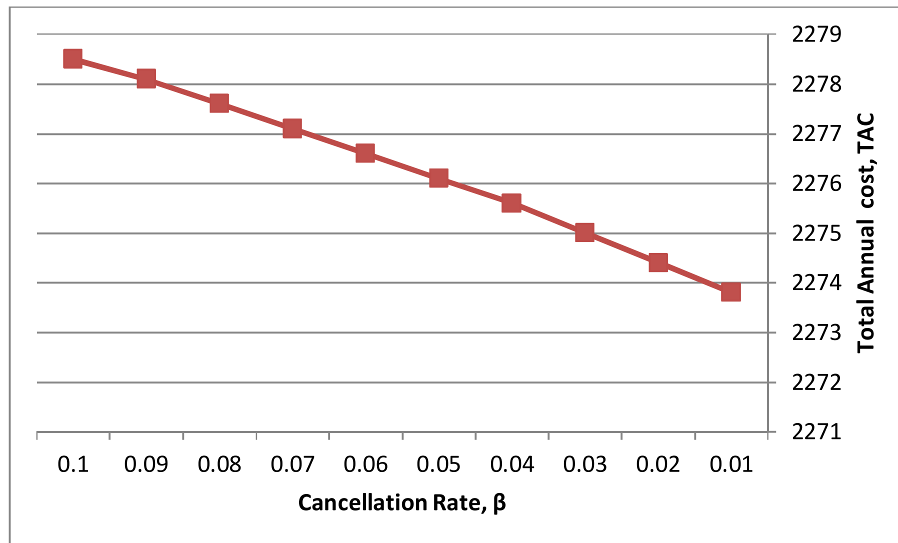


Figure 47: Effect of changing in cancellation rate on the value of TAC.

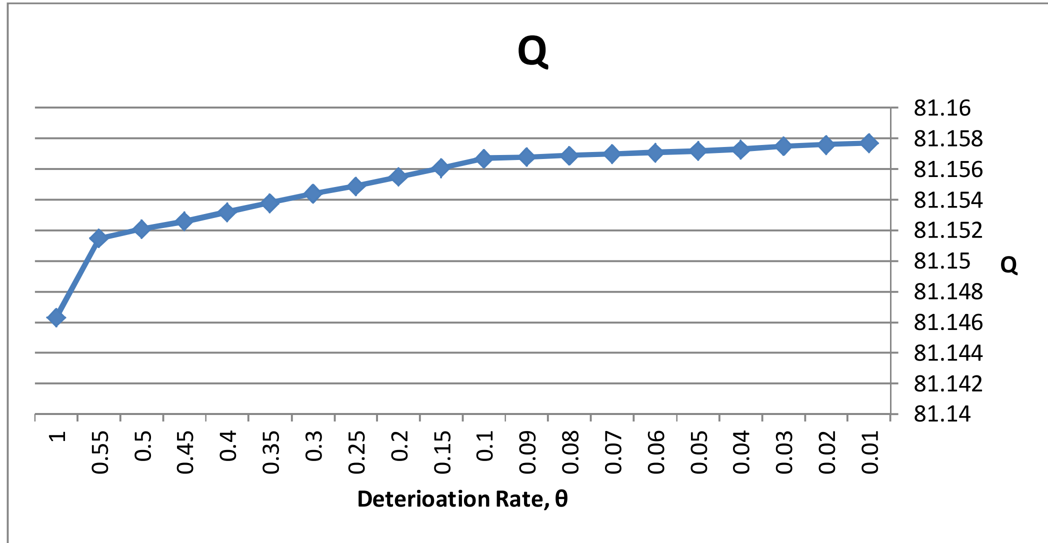


Figure 48: Effect of changing in deterioration rate on the value of Q.

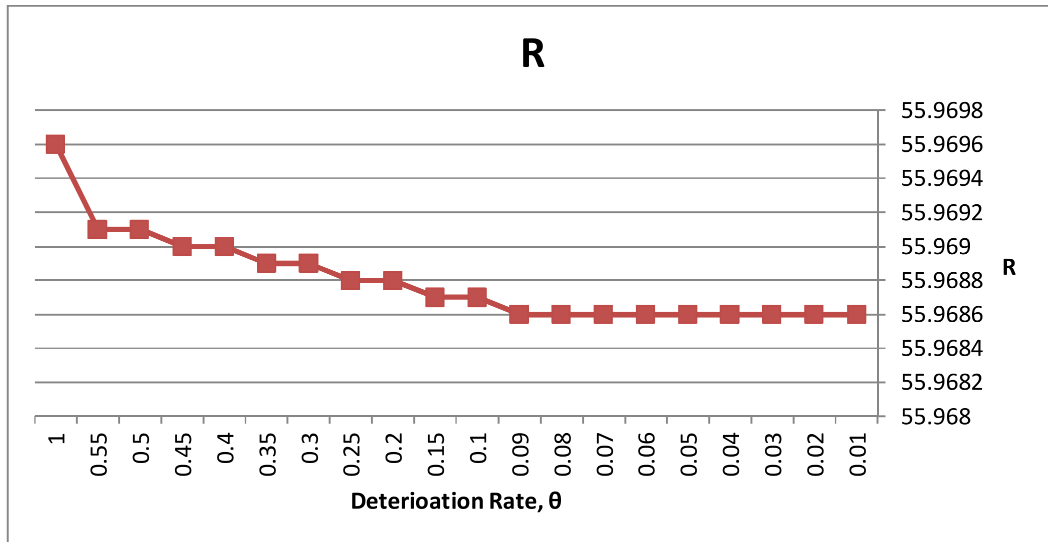


Figure 49: Effect of changing in deterioration rate on the value of R.

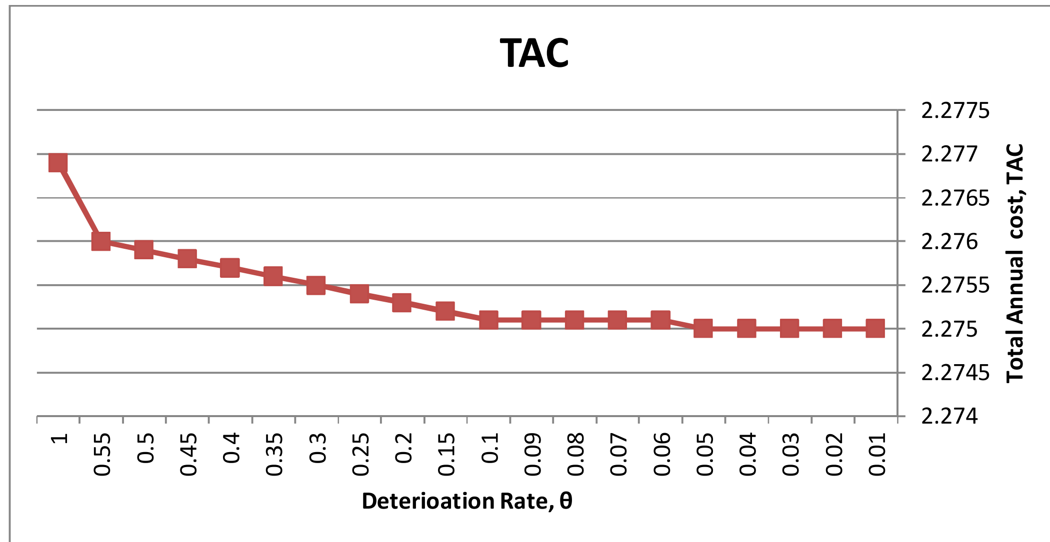


Figure 50: Effect of changing in deterioration rate on the value of TAC.

Also tornado diagram displays the above mentioned results for sensitivity analysis in Figures, 51, 52, 53, 54, 55 and 56.where the red color represents increasing effect on the variable and the blue one represents decreasing effect on the variable

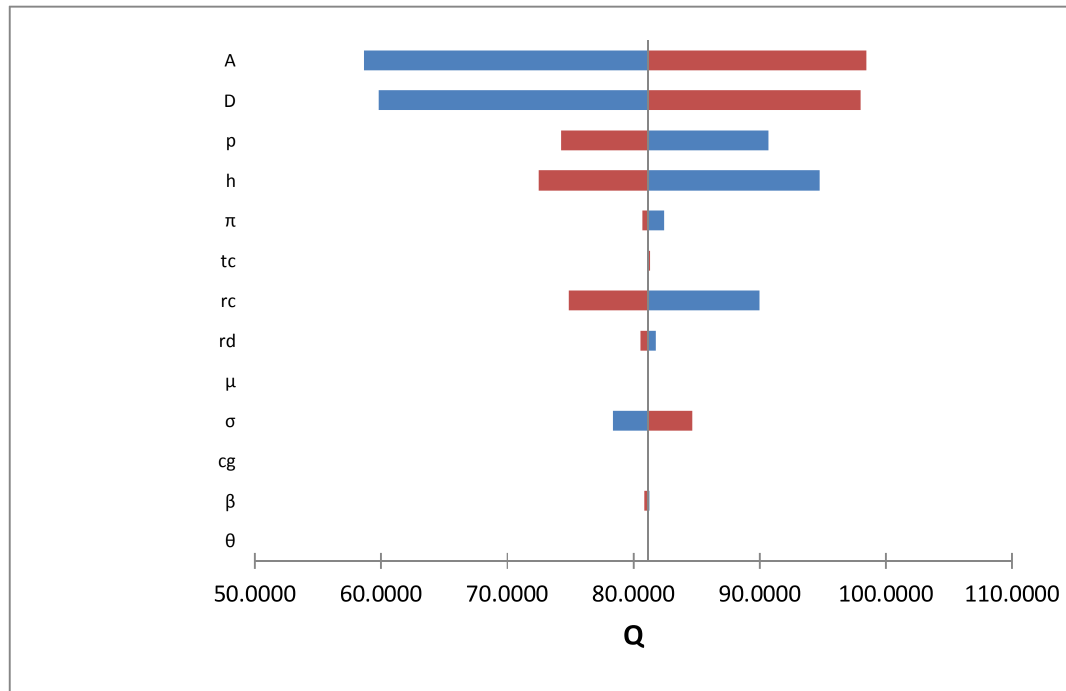


Figure 51: Tornado Diagram illustrate the effect of changing model variables on Q.

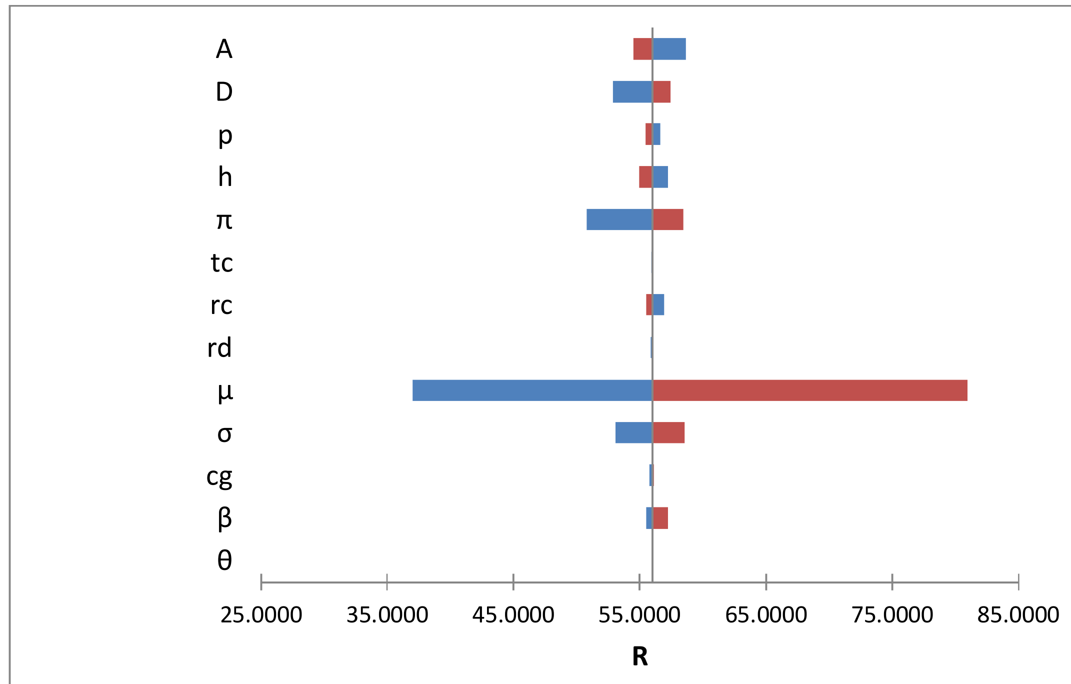


Figure 52: Tornado Diagram illustrate the effect of changing model variables on R.

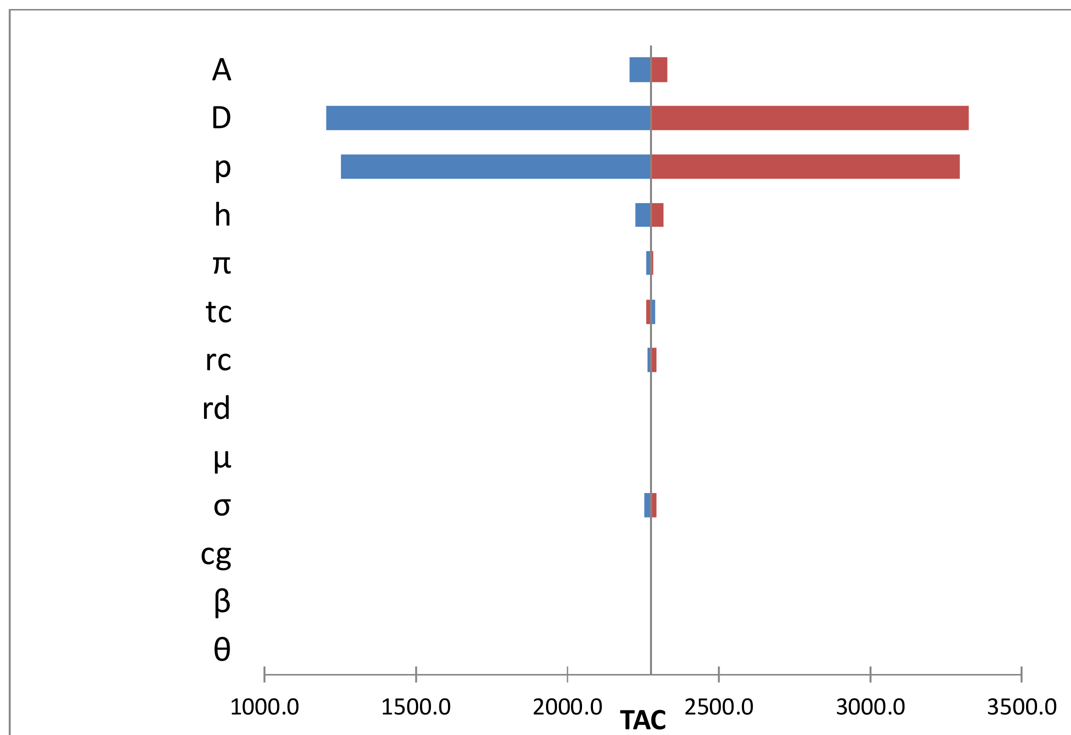


Figure 53: Tornado Diagram illustrate the effect of changing model variables on TAC.

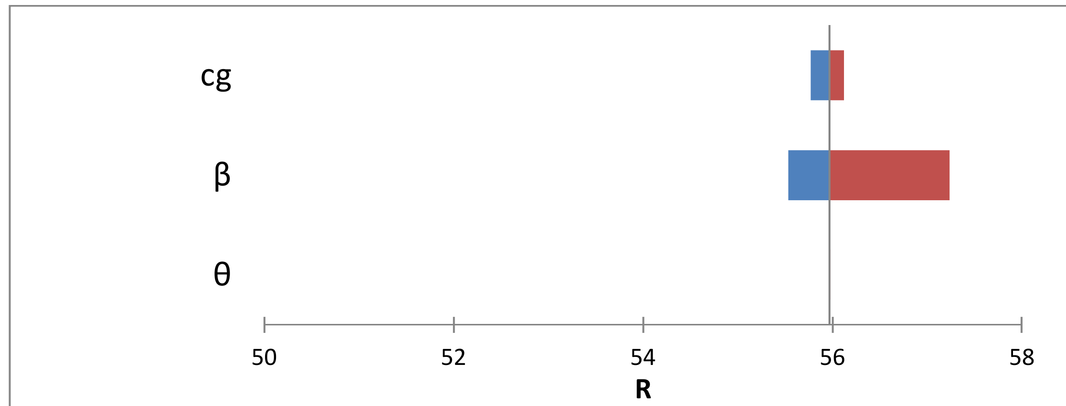


Figure 54: Tornado Diagram illustrates the effect of changing (c_g, β and θ) on R.

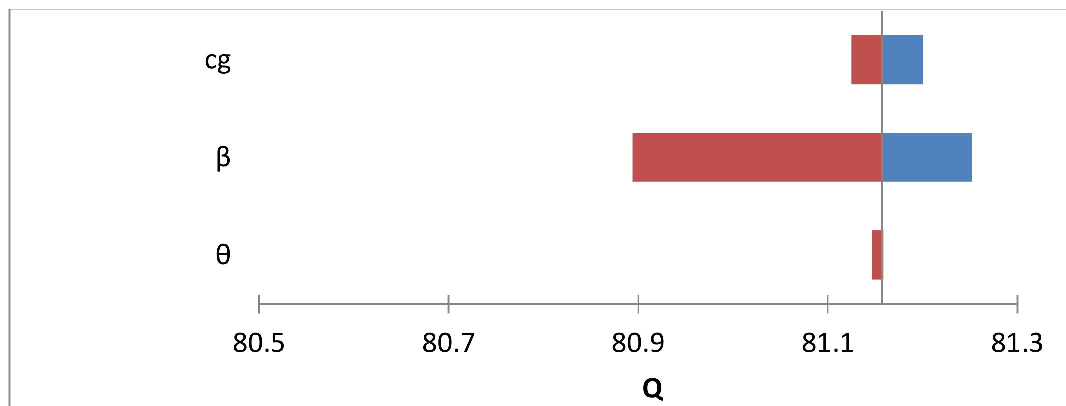


Figure 55: Tornado Diagram illustrates the effect of changing (c_g, β and θ) on Q.

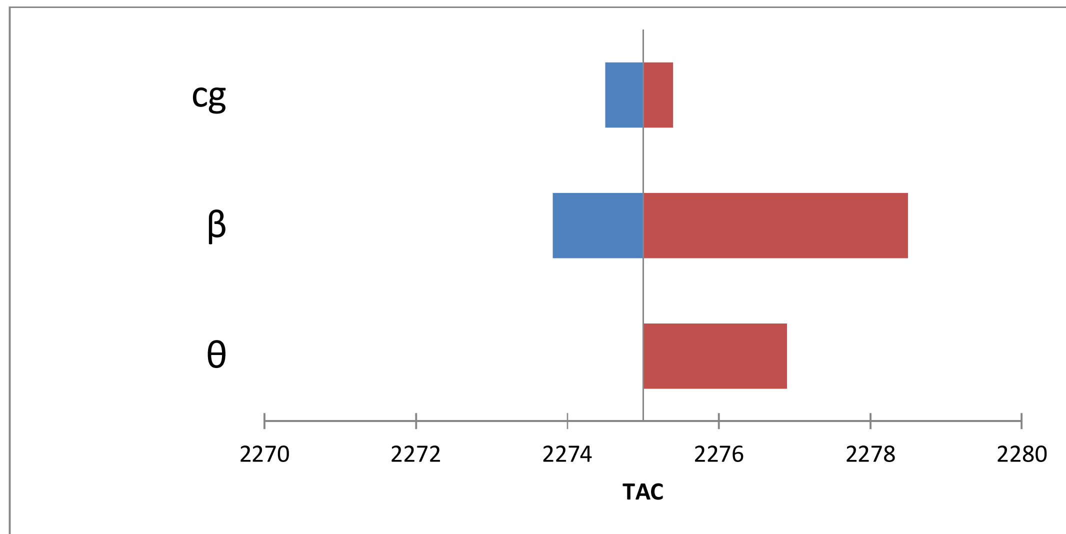


Figure 56: Tornado Diagram illustrates the effect of changing (c_g, β and θ) on TAC.

Chapter 6: Conclusions and Recommendations

The effect of credit period into continuous review inventory model for deteriorating items and back order cancellation was incorporated in this study. The lead-time demand was considered to have a normal distribution. We developed an algorithm procedure and we also used GA to obtain the optimal ordering strategy for our model, see table 1. From the table 1 we find that GA gives the larger optimal value for Q and less optimal value for R than the other algorithm but the Q value obtained from GA was equal to that one in Wu (2001) model. Also, TAC is almost the same for both algorithms but it is larger than the one obtained from Wu (2001) model. And that refers to the deterioration cost and order cancellation cost.

Furthermore, the effects of parameters on the optimal strategy were also included. From Table 3, we find that the optimal order quantity (Q) is moderately sensitive to changes in D and p, and optimal reorder point (R) has minor sensitivity to changes in the D and p. But, the minimum total annual cost TAC is highly sensitive to changes in the value of D and p.

Furthermore, to decrease the TAC and keeping Q and R constant we might change increase the interest earned from sales during credit period.

Increasing cancellation rate will increase the cost largely as shown in Table 5, for that managers must have a plane to reduce back ordering and keep low standard deviation of lead time demand in order to reduce the negative response of customers.

We find that deterioration rate has a slight effect on the values of Q, R and TAC (see table 6).

But we can see from table 3 that larger values of credit period gives lower values for Q and TAC, and that means that the customer will buy less quantity in order to take the benefits of permissible delay in payment more frequently.

And if the holding cost h is high supplier must order less quantity of inventory to keep TAC in its minimum values.

Our search may be extended to include non instantaneous deteriorating inventory where items may remain fresh for specific period of time and then starts to deteriorate. Also stochastic demand may be of interest in future search.

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Appendix 1: Genetic Algorithm

Genetic algorithms are a part of evolutionary computing, which is a rapidly growing area of artificial intelligence. They are a class of adaptive search techniques based on the principle of population genetics. GA work according to the principles of natural genetics on a population of string structures representing the problem variables. All these features make genetic algorithm search robust, allowing them to be applied to a wide variety of problems (Moghadam, et al., 2008)

5.1 Biological Background

All living organisms consist of cells. In each cell there is the same set of chromosomes. Chromosomes are strings of DNA and serves as a model for the whole organism. A chromosome consists of genes, blocks of DNA. Each gene encodes a particular protein. Basically, it can be said, that each gene encodes a trait, for example color of eyes. Possible settings for a trait (e.g. blue, brown) are called alleles. Each gene has its own position in the chromosome. This position is called locus. Complete set of genetic material (all chromosomes) is called genome. Particular set of genes in genome is called genotype. The genotype is with later development after birth base for the organism's phenotype, its physical and mental characteristics, such as eye color, intelligence etc.

During reproduction, first occurs recombination (or crossover). Genes from parents form in some way the whole new chromosome. The new created offspring can then be mutated. Mutation means, that the elements of DNA are a bit changed. These changes are mainly caused by errors in copying genes from parents. And the fitness of an organism is measured by success of the organism in its life.

5.2 Search Space

As in Raulins (1991) when solving some problem, you are usually looking for some solution, which will be the best among others. The space of all feasible solutions (it means objects among those the desired solution is) is called search space (also state space). Each point in the search space represents one feasible solution. Each feasible solution can be "marked" by its value or fitness for the problem. We are looking for our solution, which is one point (or more) among feasible solutions - that is one point in the search space.

The looking for a solution is then equal to a looking for some extreme (minimum or maximum) in the search space. The search space can be whole known by the time of solving a problem, but usually you know only a few points from it and we are generating other points as the process of finding solution continues. The problem is that the search can be very complicated. One does not know where to look for the solution and where to start. There are many methods, how to find some suitable solution (ie. not necessarily the best solution), for example hill climbing, tabu search, simulated annealing and genetic algorithm. The solution found by these methods is often considered as a good solution, because it is not often possible to prove what the real optimum is.

5.3 Basic Description Genetic Algorithm

Genetic algorithms are inspired by Darwin's theory about evolution. Solution to a problem solved by genetic algorithms is evolved. Algorithm is started with a set of solutions represented by chromosomes called population. Solutions from one population are taken and used to form a new population. This is motivated by a hope, that the new population will be better than the old one. Solutions which are selected to

form new solutions offspring are selected according to their fitness; the more suitable they are the more chances they have to reproduce. This is repeated until some condition for example number of populations or improvement of the best solution is satisfied.

Outline of the Basic Genetic Algorithm

Das, et al. (2010) and Huapt and Huapt (2004) described the steps of genetic algorithm which is:

[Start] **Generate random population of *n* chromosomes (suitable solutions for the problem)**

1. **[Fitness]** Evaluate the fitness $f(x)$ of each chromosome x in the population
2. **[New population]** Create a new population by repeating following steps until the new population is complete
    1. **[Selection]** Select two parent chromosomes from a population according to their fitness (the better fitness, the bigger chance to be selected)
    2. **[Crossover]** With a crossover probability cross over the parents to form a new offspring (children). If no crossover was performed, offspring is an exact copy of parents.
    3. **[Mutation]** With a mutation probability mutate new offspring at each locus (position in chromosome).
    4. **[Accepting]** Place new offspring in a new population
3. **[Replace]** Use new generated population for a further run of algorithm
4. **[Test]** If the end condition is satisfied, **stop**, and return the best solution in current population
5. **[Loop]** Go to step **2**

5.4 Operators of GA

As you can see from the genetic algorithm outline, the crossover and mutation are the most important part of the genetic algorithm. The performance is influenced mainly by these two operators.

A. Crossover

After decision of the type of encoding to be used a step to crossover is made. As discussed in Das, et al. (2010) Crossover selects genes from parent chromosomes and creates a new offspring. The simplest way how to do this is to choose randomly some crossover point and everything before this point copy from a first parent and then everything after a crossover point copy from the second parent. (See Figure 57) where | is the crossover point:

Chromosome 1	11011 | 00100110110
Chromosome 2	11011 | 11000011110
Offspring 1	11011 | 11000011110
Offspring 2	11011 | 00100110110

Figure 57: crossover in GA.

There are other ways how to make crossover, for example we can choose more crossover points as Beasley et, all (1993) he discussed that Crossover can be rather complicated and very depends on encoding of the encoding of chromosome. Specific crossover made for a specific problem can improve performance of the genetic algorithm.

B. Mutation

After a crossover is performed Das, et al. (2010) start mutation which is important to prevent falling all solutions in population into a local optimum of solved

problem. Mutation changes randomly the new offspring. For binary encoding we can switch a few randomly chosen bits from 1 to 0 or from 0 to 1. Mutation can then be as the example in Figure 58:

Original offspring 1	1101111000011110
Original offspring 2	1101100100110110
Mutated offspring 1	1100111000011110
Mutated offspring 2	1101101100110110

Figure 58: Mutation in GA.

The mutation depends on the encoding as well as the crossover. For example when we are encoding permutations, mutation could be exchanging two genes Das et,all. (2010).

5.5 Parameters of GA

There are two basic parameters of GA - crossover probability and mutation probability. Crossover probability says how often will be crossover performed. If there is no crossover, offspring is exact copy of parents. If there is a crossover, offspring is made from parts of parents' chromosome. If crossover probability is 100%, then all offspring is made by crossover. If it is 0%, whole new generation is made from exact copies of chromosomes from old population (but this does not mean that the new generation is the same!). Crossover is made in hope that new chromosomes will have good parts of old chromosomes and maybe the new chromosomes will be better. However it is good to leave some part of population survive to next generation Das, et al. (2010).

Mutation probability says how often will be parts of chromosome mutated. If there is no mutation, offspring is taken after crossover (or copy) without any change. If mutation is performed, part of chromosome is changed. If mutation probability is 100%, whole chromosome is changed, if it is 0%, nothing is changed. Mutation is made to prevent falling GA into local extreme, but it should not occur very often, because then GA will in fact change to random search (Das,et al., 2010).

There are also some other parameters of GA. One also important parameter is population size. Population size says how many chromosomes are in population (in one generation). If there are too few chromosomes, GA has a few possibilities to perform crossover and only a small part of search space is explored. On the other hand, if there are too many chromosomes, GA slows down. After some limit (which depends mainly on encoding and the problem) it is not useful to increase population size, because it does not make solving the problem faster.

5.6 Selection

As from the GA outline, chromosomes are selected from the population to be parents to crossover. The problem is how to select these chromosomes. According to Darwin's evolution theory the best ones should survive and create new offspring. There are many methods how to select the best chromosomes, for example uniform selection random selection, roulette wheel selection and tournament selection (Das,et al., 2010).

A. (Uniform selection)Pairing from top to bottom

Start at the top of the list and pair the chromosomes two at a time until the top chromosomes are selected for mating. Thus, the algorithm pairs odd rows with even rows. The mother has row numbers in the population matrix and the father has the row

numbers, this approach doesn't model nature well but is very simple to program (Haupt and Haupt, 2004).

B. Random pairing

This approach uses a uniform random number generator to select chromosomes (Haupt and Haupt, 2004).

C. Roulette Wheel Selection (Weighted random pairing)

The probabilities assigned to the chromosomes in the mating pool are inversely proportional to their cost. A chromosome with the lowest cost has the greatest probability of mating, while the chromosome with the highest cost has the lowest probability of mating. A random number determines which chromosome is selected (Haupt and Haupt, 2004), (See Figure 59).

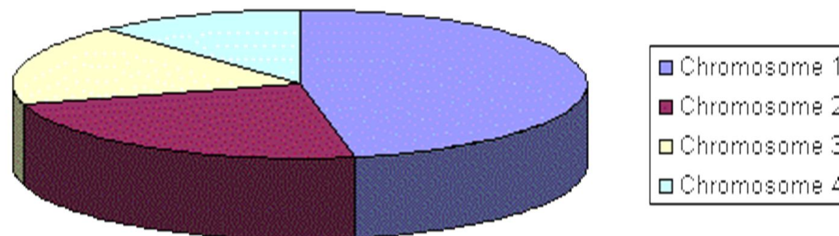


Figure 59: Roulette Wheel Selection Method in GA.

Then a marble is thrown there and selects the chromosome. Chromosome with bigger fitness will be selected more times. This can be simulated by following algorithm.

1. **[Sum]** Calculate sum of all chromosome fitness in population - sum ***S***.
2. **[Select]** Generate random number from interval ***(0,S)*** - ***r***.
3. **[Loop]** Go through the population and sum fitness from ***0*** - sum ***s***. When the sum ***s*** is greater than ***r***, stop and return the chromosome where you are.

Of course, step **1** is performed only once for each population.

As in (Haupt and Haupt, 2004) there are two techniques for Roulette selection which are Rank selection and cost selection:

a) Rank Selection

The previous selection will have problems when the fitness differs very much. For example, if the best chromosome fitness is 90% of the entire roulette wheel then the other chromosomes will have very few chances to be selected, but Rank selection is problem independent and finds the probability from the rank, n, of the chromosome, first ranks the population and then every chromosome receives fitness from this ranking. The worst will have fitness 1, second worst 2 etc. and the best will have fitness N (number of chromosomes in population). As you can see in Figure 60 and Figure 61, how the situation changes after changing fitness to order number:

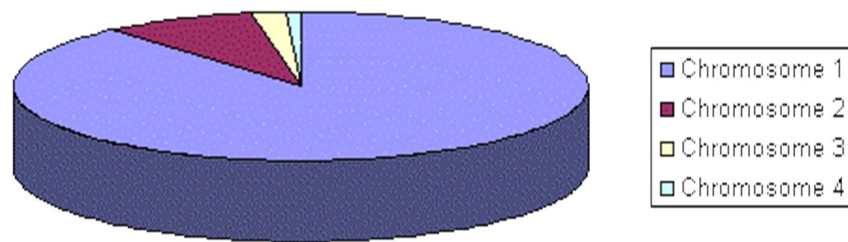


Figure 60: Situation before ranking (graph of fitness) in GA.

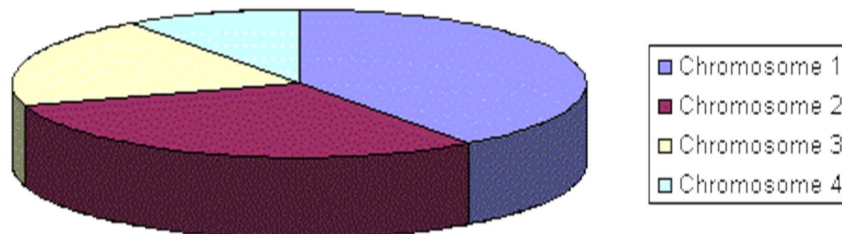


Figure 61: Situation after ranking (graph of order numbers) in GA.

After this all the chromosomes have a chance to be selected. But this method can lead to slower convergence, because the best chromosomes do not differ so much from other ones

b) Cost selection

The probability of selection is calculated from the cost of the chromosome rather than its rank in the population. A normalized cost is calculated for each chromosome by subtracting the lowest cost of the discarded chromosomes ( ) from the cost of all the chromosomes in the mating pool. This approach tends to weight the top chromosome more when there is a large spread in the cost between the top and bottom chromosome. On the other hand, it tends to weight the chromosomes evenly when all the chromosomes have approximately the same cost. The same issues apply as discussed above if a chromosome is selected to mate with itself. The probabilities must be recalculated each generation.

D. Tournament selection

Another approach that closely mimics mating competition in nature is to randomly pick a small subset of chromosomes (two or three) from the mating pool, and the chromosome with the lowest cost in this subset becomes a parent. The tournament repeats for every parent needed. Threshold and tournament selection make a nice pair, because the population never needs to be sorted. Tournament selection works best for larger population sizes because sorting becomes time-consuming for large populations.

5.7 Encoding

Encoding of chromosomes is one of the problems, when you are starting to solve problem with GA. Encoding very depends on the problem. The chromosome

should in some way contain information about solution which it represents. The most used way of encoding is a binary string. The chromosome then could look like this:

Chromosome 1	1101100100110110
Chromosome 2	1101111000011110

Each chromosome has one binary string. Each bit in this string can represent some characteristic of the solution. Or the whole string can represent a number - this has been used in the basic **GA applet**. Of course, there are many other ways of encoding. This depends mainly on the solved problem. For example, one can encode directly integer or real numbers; sometimes it is useful to encode some permutations and so on. But Binary Encoding will be discussed next.

Binary encoding is the most common, mainly because first works about GA used this type of encoding. In **binary encoding** every chromosome is a string of **bits**, **0** or **1**. The following is an example for binary encoding of chromosomes:

Chromosome A	101100101100101011100101
Chromosome B	111111100000110000011111

Binary encoding gives many possible chromosomes even with a small number of alleles. On the other hand, this encoding is often not natural for many problems and sometimes corrections must be made after crossover and/or mutation.

5.8 Crossover and Mutation for binary encoding methods

Crossover and mutation are two basic operators of GA. Performance of GA very depend on them. Type and implementation of operators depends on encoding and also on a problem. There are many ways how to do crossover and mutation. Following is only some examples and suggestions how to do it for several encoding:

Single point crossover – as in Beasley, et al. (1993) one crossover point is selected, binary string from beginning of chromosome to the crossover point is copied from one parent and the rest is copied from the second parent, (see Figure 62).

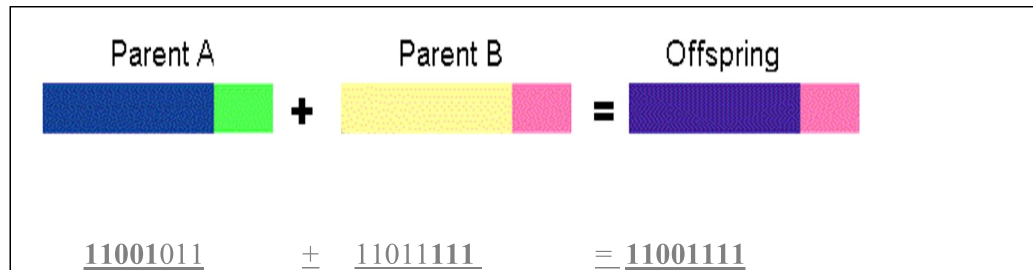


Figure 62: An example of Single point crossover in binary encoding of chromosomes.

Two point crossover – Also Beasley, et al. (1993) studied the case of two crossover point in binary encoding where two crossover points are selected, binary string from beginning of chromosome to the first crossover point is copied from one parent, the part from the first to the second crossover point is copied from the second parent and the rest is copied from the first parent, (see Figure 63).

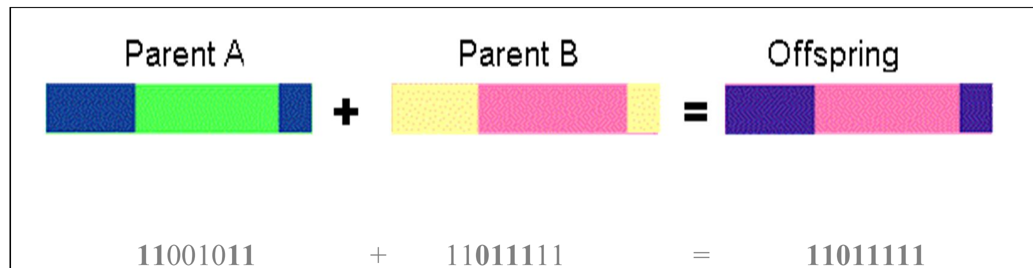


Figure 63: An example of two point crossover in binary encoding of chromosomes.

Uniform crossover – in this type of crossover in binary encoding as discussed in Beasley, et al. (1993), bits are randomly copied from the first or from the second parent, as Shown in Figure (64).

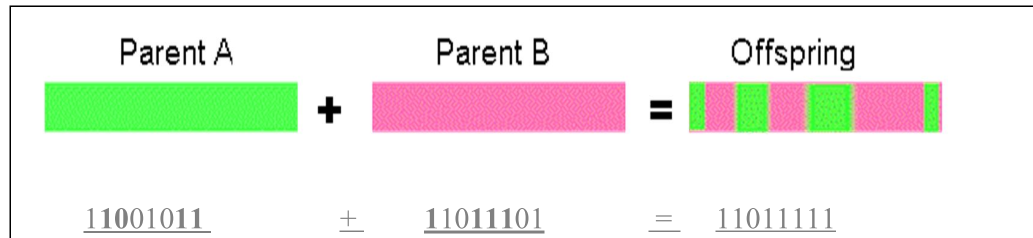


Figure 64: An example of uniform crossover in binary encoding of chromosomes.

Bit inversion Mutation where selected bits are inverted, as shown in Figure 65.

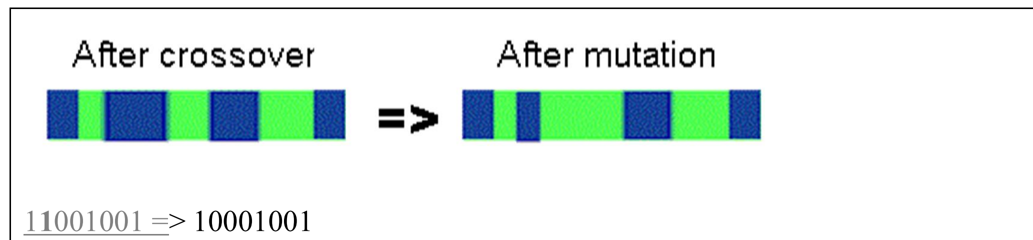


Figure 65: An example of Bit inversion Mutation in binary encoding of chromosomes.

Appendix 2:

MATLAB cod for simple to use algorithm:

```
Q0 =100
 Q=10
A =50
 D =200
 p = 10
h = 2
by = 5
tc = 0.1
rc = 0.15
rd = 0.12
mu1 = 50
sigma1=9
cg =6
beta = 0.03
theta = 0.03
x0=0;
k0=10;
k=100;
W=[];
E=[];
while abs(k-k0)>0.001 || abs(Q-Q0)>0.001
phi_k = ((Q0*(h+p*rc)/((by*D)+(beta*(p+cg)*D)+(p*tc*rd*D))))
k0=k
F=@(x)1-normcdf(x)-phi_k
[x,fval] = fzero(F,x0)
k=x
x0=x
R = mu1 + k*sigma1
n_R = sigma1*(normpdf(k)-k*(1-normcdf(k)));
Q0=Q
Q = abs(sqrt((A*D + (((D-theta)^2)*p*rc*(tc^2)/2) + by*n_R*D + ...
```

```
n_R*beta*(p+cg)*D-((D^2)*p*(tc^2)*rd/2)-n_R*p*tc*rd*D)/((h+p*rc)/2)))
W=[W;Q]
ordering_cost = (A*D/Q) + (p*D);
cycle_inventory_cost = h*Q/2;
ICOHI_tc = ((Q - D*tc - theta*tc)^2)*p*rc/(2*Q);%Interest Charged for
%On Hand Inventory @ tc
safety_stock_cost = (h + p*rc)*(R - mu1-(theta*tc));
shortage_cost = by*n_R*D/Q;
lost_sales_cost = n_R*beta*(p + cg)*D/Q;
IEFDSDCP = -(D^2)*p*(tc^2)*rd/(2*Q);     %Interest Earned From Demand
%Sales During Credit Period
IEFSBFPP = -n_R*p*tc*rd*D/Q;             %Interest Earned From Sales
                                         %of Backorders From Previous
                                         %Period
deteritation_cost=p*theta*tc*D/Q
 summm = ordering_cost + cycle_inventory_cost + ICOHI_tc + ...
     safety_stock_cost + shortage_cost + lost_sales_cost + ...
     IEFDSDCP + IEFSBFPP + deteritation_cost

E=[E;summm]
end
```

Appendix 3:

MATLAB cod for Genetic Algorithm Cod:

To validate our model using Genetic Algorithm we used the) gatool in MATLAB as following:

1. Population type is double vector.
2. Population size is the default and equal to 20.
3. Creation Function is constraint dependent.
4. Initial score is default.
5. Initial range is [0, 1].
6. Scaling function is Rank.
7. Selection function is stochastic uniform.
8. Elite count for reproduction is 2.
9. Crossover function is 0.8 and Scattered.
10. Direction of mutation is forward, with fraction equal to 0.2 and interval equal to 20.
11. **For Stopping Criteria:**
    a. **Generation is 100.**
    b. **Time limit is infinite.**
    c. **Fitness limit is infinite.**
    d. **Stall generation is 50**
    e. **Function tolerance is e^{-6}**

نموذج ضبط المخزون القابل للهلاك بتحديد كمية الطلب و نقطة الطلب في حال السماح بتاخير الدفعات و الغاء الطلبات المعلقة.

اعداد

ديمة عبدالله محمد عايش

المشرف

الدكتور محمود برغش

ملخص

مراقبة المخزون عملية مهمة لضمان مراقبة الجودة في الشركات التي تتعامل مع السلع الاستهلاكية. و نظام مراقبة المخزون الفعال يقوم بالتنبيه عندما يحين الوقت لطلب البضاعة وذلك لتفادي نفاد المخزونات ، لمنع نقص الكميات بسبب التلف أو السرقة ، وتوفير محاسبية سليمة.في هذه الدراسة قمنا بوضع نموذج ضبط المخزون القابل للهلاك بتحديد كمية الطلب و نقطة الطلب في حال السماح بتاخير الدفعات و الغاء الطلبات المعلقة. حيث تم تطوير النموذج المعد في Wu (2001) ليشمل المخزون القابل للهلاك والغاء الطلبات المعلقة.

الهدف من الدراسةايجاد افضل قيمة لكمية الطلب و افضل قيمة لنقطة اعادة الطلب بحيث تكون التكلفة الاجماليةالسنوية بحدها الادنى. وقد تم تطوير نموذج رياضي لحل هذه المشكلة، ثم تم استخدام طريقتين لحل نموذج المطوروهما خوارزمية بسيطة والخوارزمية الجينية. تم استخدام برنامج MATLABللعثور على نتائج الخوارزميتين. وتم التحقق من صحة النموذج باستخدام المثال المستخدم في Wu(2001). وأجريت أيضا تحليل الحساسية لدراسة تأثير عوامل النموذج على (قيمة كمية الطلب و قيمة نقطة اعادة الطلب). ووفقا لنتائج المثال العددي تم تقديم بعض التوصيات التي تساعد على التقليل التكلفة السنوية الاجمالية للمخزون.

بمقارنة نتائج الخوارزميتين المستخدمتين لحل المشكلة (الخوارزمية البسيطة والخوارزمية الجينية)، ظهر أن الخوارزمية الجينية تعطي قيمة أقل لكمية الطلب و لنقطة اعادة الطلب لكنها تعطي نفس القيمة للكلفة السنوية الاجمالية للمخزون. و بناءً على تحليل الحساسية تبين أن كمية الطلب المثلى (Q) حساسة للتغيرات في متوسط الطلب السنوي (D) و تكلفة الشراء للوحدة الواحدة (p)، اما بالنسبة لنقطة إعادة الطلب المثلى (R) فهي أقل حساسية للتغيرات في متوسط الطلب السنوي (D) و تكلفة الشراء للوحدة الواحدة (p)، اما الحد الادنى لقيمة الكلفة الاجمالية السنوية للمخزون فهي شديدة الحساسية للتغيرات متوسط الطلب السنوي (D) و تكلفة الشراء للوحدة الواحدة (p). وعلاوة على ذلك لتخفيض الكلفة الاجمالية السنوية للمخزون مع الحفاظ قيمة كمية الطلب و قيمة نقطة اعادة الطلب ثابتتين يجب زيادة الفوائد المتحصلة من المبيعات خلال فترة السماح بـاخير الدفعات. كما وان الزيادة على معدل الهلاك له تأثير طردي بسيط على الكلفة السنوية الاجمالية للمخزون، وبالمقابل فإن الزيادة على كلفة فقدان الزبون والزيادة على معدل الغاء الطلبات يؤدي الى زيادة ملحوظة على الكلفة الاجمالية للمخزون.